A Spectrally Efficient MIMO System with Sparse Matrix Precoding

by

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Declaration

I hereby declare that

i) the thesis comprises of my original work towards the degree of Master of Technology in Electronics and Communications at Dhirubhai Ambani Institute of Information and Communication Technology & C.R.Rao Advanced Institute of Applied Mathematics, Statistics and Computer Science, and has not been submitted elsewhere for a degree,

ii) due acknowledgment has been made in the text to all the reference material used.

Pyadav

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Certificate

This is to certify that the thesis work entitled A Spectrally Efficient MIMO System with Sparse Matrix Precoding has been carried out by Prabhanshu Yadav for the degree of Master of Technology in Electronics and Communications at *Dhirubhai Ambani Institute of Information and Communication Technology & C.R.Rao Advanced Institute of Applied Mathematics, Statistics and Computer Science* under my/our supervision.

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Thesis Supervisor

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Contents

Abstract

This thesis proposes a novel technique of sparse matrix-based precoding at the transmitter of a Multiple Input Multiple Output (MIMO) system. We proposed two sparse matrix precoded MIMO systems. Our first proposal improves the spectral efficiency beyond the existing spectral efficiency of Precoding-aided Spatial Modulation (PSM-MIMO) system. Our second proposal increases spectral efficiency compared to an existing MIMO system.

Both proposals use a two-stage precoding approach in which the conventional zero-forcing (ZF) MIMO precoder, which inverts the matrix MIMO channel, is combined with a sparse matrix precoding. With the conventional ZF precoder, the degrees of freedom (DoF) available at the transmitter equals the number of antennas at the receiver. By adding another layer of precoding using a sparse matrix, we increase the DoF at the transmitter, thereby facilitating an increase in spectral efficiency. We demonstrate proof of the concept (PoC) by simulation-driven experiments. Our PoC is based on the ML (Maximum Likelihood) detection at the receiver. ML detection has quite high complexity. We propose a belief propagation algorithm at the receiver which is more practical to implement in a real-world system. The belief propagation algorithm leverages the sparseness of the precoding matrix and has low computational complexity.

Keywords: *SMP-MIMO, SMP-PSM-MIMO, spectral efficiency, zero-forcing precoder, ML Detector, LDPC, belief propagation.*

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CHAPTER 1

Introduction

1.1 Motivation

The 5G (and beyond) systems leverage advances in multiple domains of the wireless communications theory/practice to meet increasing requirement for higher throughput and better QoS (quality of service). For 5G (and beyond) systems, larger bandwidth, higher throughput and better QoS are not the only requirement. As we move forward to the next generation wireless communication, it will have new and latest applications such as machine to machine (M2M) communication, smart home appliances and internet of things (IoT) [1]. For all these application, 5G and beyond system need to be Ultra-reliable Low-Latency Communication (URLLC). 5G (and beyond) systems also need to be energy efficient. Keeping all this in mind researchers have proposed massive Multiple Input Multiple Output (mMIMO), filter bank multi carrier (FBMC) modulation, millimeter-wave (mm-wave) communication, and non-orthogonal multiple access (NOMA) technologies. These technologies have been considered to have strong potential to fulfill the requirement of 5G (and beyond) communication system [2]. From the energy efficiency perspective, there have been a lot of talk about the index modulation (IM) in the literature. Index modulation is considered an energy-efficient solution for 5G (and beyond) communication.

MIMO technology has greatly increased the throughput and reliability of wireless communication. The spatial multiplexing using MIMO (SMUX-MIMO) where all transmitting antennas transmit simultaneously and over the same spectral band send one of M symbols from a M-ary Amplitude and Phase Modulation (APM). For SMUX-MIMO, there is a requirement of carefully mitigating the MAI – the multi-antenna (co-channel) interference [3]. The SMUX-MIMO either addresses this task by utilizing the channel state information (CSI) either at the receiver (CSIR) or at the transmitter (CSIT). An advantage of the latter is that the receiver complexity is reduced [4].

Index modulation is one of the MIMO techniques where the indices of the antennas at the transmitter or at the receiver are used to transmit the additional information. When we use the indices of the transmit antenna to transmit extra information in addition to the M -ary APM constellations, we call it transmit spatial modulation or which is also termed as spatial modulation (SM) MIMO system [5, 6]. A MIMO technique that avoids the MAI problem while offering a reduced implementation complexity compared to that of the SMUX-MIMO. In SM-MIMO, only one of N_T transmit antennas is activated over a symbol duration. SM-MIMO uses the CSI at the receiver, with the resultant implementation cost at the receiver.

When we use the indices of the antenna at the receiver to transmit additional information in addition to the conventional M -ary APM constellations, we call it the spatial modulation at the receiver (RSM), which is also termed as Precoding-Aided Spatial Modulation (PSM) MIMO system [7]. In PSM-MIMO, only one of N_T receive antennas is activated over a symbol duration. PSM-MIMO system uses CSIT, which helps in the reduction of the complexity of the receiver. The technique of zero-forcing (ZF) transmit precoding is employed with CSIT in PSM-MIMO [7].

The maximum realizable spectral efficiency of the SMUX-MIMO equals $N_T \times \log_2(M)$ bits per channel use (bpcu), which exceeds the APM spectral efficiency of $\log_2(M)$ bpcu by a factor of N_T. SM-MIMO provides the spectral efficiency of $\log_2(N_T \times M)$ bpcu and PSM-MIMO provides the spectral efficiency of $\log_2(N_R \times M)$ bpcu. Despite having numerous advantages of SM-MIMO and PSM-MIMO. The drawback of both schemes is their spectral efficiency. The throughput (spectral efficiency) of SM-MIMO and PSM-MIMO is smaller than the SMUX-MIMO.

Different techniques have been proposed in the literature to increase the throughput of SM-MIMO and PSM-MIMO systems. Generalised spatial modulation (GSM) MIMO is one technique that allows us to go beyond the limitation of SM-MIMO that the number of transmit antenna required has to be a power of two [8]. In GSM-MIMO, the extra information is mapped to the number of transmit antennas activated at a time. GSM-MIMO achieves higher spectral efficiency by the base-two logarithm of the number of activated antenna combinations. This helps in the reduction of the total number of transmit antennas required to achieve the same spectral efficiency in SM-MIMO [8]. Unlike GSM, which is performed at the transmitter, if we perform GSM at the receiver side then it is reffered to as generalised PSM (GPSM) MIMO system [9, 10]. GPSM-MIMO has the same benefits as GSM-MIMO with respect to spectral efficiency. Also, in PSM-MIMO the receiver complexity is less because it is operated under CSIT mode. Therefore, the receiver compelexity of GPSM-MIMO would also be less compared to GSM-MIMO.

1.2 Contributions

In this thesis, we present a novel technique – the Sparse Matrix Precoded MIMO (SMP-MIMO). Our first proposal targets to increase the spectral efficiency beyond the existing spectral efficiency of the PSM-MIMO system where as our second proposal aims at increasing the spectral efficiency beyond the existing spectral efficiency of the MIMO system without increasing the number of transmit antennas or received antennas in the given MIMO system.

In the proposed schemes, we use two layers of precoding at the transmitter of a MIMO system. The novelty of the proposed system is the first layer of precoding, i.e., an $\boldsymbol{A}$ matrix, which has not been introduced in the literature yet. With the help of this $\boldsymbol{A}$ matrix, we are increasing the spectral efficiency in our proposed schemes. The dimension of $\boldsymbol{A}$ matrix are $N_R \times N_T^{'}$ where $N_T^{'}$ » N_T and N_R. This $\boldsymbol{A}$ matrix is a sparse matrix carefully designed to reduce the interference among the elements of the information-bearing vector. The sparsity of $\boldsymbol{A}$ matrix could be further leveraged to reduce the computational complexity.

The second layer is a ZF matrix that inverts the MIMO channel matrix. Our first proposal is analogous to PSM-MIMO and the second proposal is similar to the ZF-precoded-MIMO except for an added layer of precoding. Our proposed system utilizes CSI at the transmitter and helps reduce the receiver complexity. When we have perfect CSIT, a ZF precoding matrix is obtained by the pseudo-inverse of the channel matrix.

In our proposed systems, the degrees of freedom (DoF) available at the transmitter equals to the number of columns of the $\boldsymbol{A}$ matrix. The spectral efficiency of the first proposal is $\log_2(N_T^{'} \times M)$, which is greater than the spectral efficiency of the PSM-MIMO system. The spectral efficiency of the second proposal is $N_T^{'}$ x $\log_2(M)$ bpcu, which is greater than the spectral efficiency of the MIMO system.

At the receiver, two methods are used for decoding. Maximum Likelihood (ML) decoder and Belief Propagation algorithm-based decoder. To counteract the high computational complexity of the ML decoder, we used the belief propagation algorithm at the receiver to leverage the sparsity of $\boldsymbol{A}$ matrix. The Belief propagation algorithm has low computational complexity and is more practical to implement at the receiver. We have shown the theoretical analysis for the SER expression of the proposed SMP-MIMO scheme.

1.3 Thesis Organisation

In Chapter two, contains the discussion of the background for the problem statement. Chapter three contains the formulation of the problem statement. Chapter four, contains the literature survey where we discuss the existing ways to solve the research problem. In Chapter five, we discuss the proposed solution for the given problem statement in chapter three. After that, we move to chapter six which contains the conclusion for the proposed solution. Chapter seven contains the possible future work for the proposed problem and the solution. Chapter eight contains the references for this thesis.

CHAPTER 2
Background

2.1 MIMO

As we go into next generation wireless communication system, it is prominent that a higher data rate will be required for wireless communication. To achieve the multiplicative increase in data rate, we can use multiple number of transmit antennas and receive antennas. This usage of multiple number of transmit and receive antenna system is also known as MIMO system [11]. The multiplicative increase in spectral efficiency of the MIMO system is with respect to a single stream, also known as the SISO system. In MIMO, the increase in spectral efficiency is achieved through multiplexing. MIMO not only provides gain in spectral efficiency but also helps in the improvement of the performance of the system by the virtue of diversity gain.

2.1.1 MIMO System Model

Take into consideration a MIMO system that consists of N_R receive antennas and N_T transmit antennas as shown in Figure 2.1. The following equation can express the MIMO system

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{s} + \boldsymbol{n}, \tag{2.1}$$

where $\boldsymbol{y}$ is a N_R dimensional received vector at the receiver. MIMO channel matrix is denoted by $\boldsymbol{H}$ matrix of dimension N_R x N_T and $\boldsymbol{x}$ is N_T dimensional information bearing vector. The transmit vector $\boldsymbol{s}$ is same as the information-bearing vector $\boldsymbol{x}$ so, the above equation can also be written as

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{n} \tag{2.2}$$

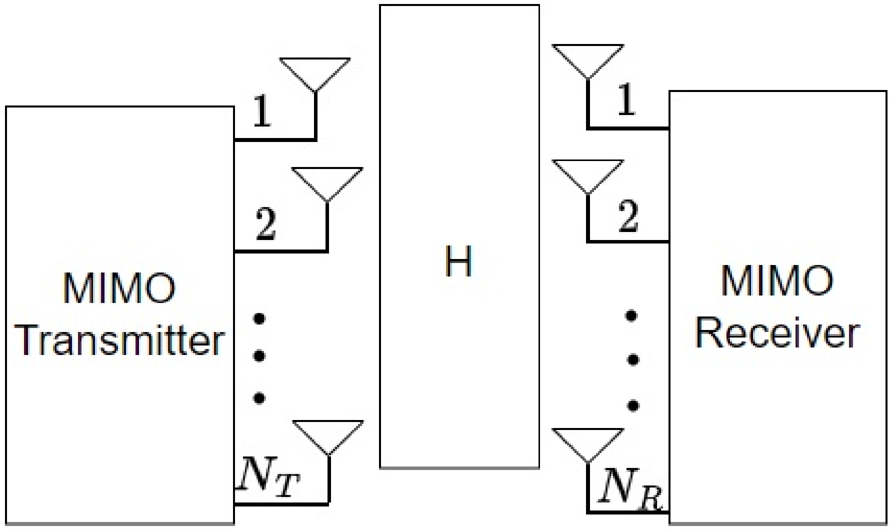

Figure 2.1: MIMO system model.

The spectral efficiency of a SISO system is $\log_2(M)$ bpcu. The SMUX-MIMO is when all N_T transmit antennas transmit simultaneously and over the same spectral band send one of M symbols from a M-ary APM constellations, the maximum realizable spectral efficiency of the SMUX-MIMO system equals $N_T \times \log_2(M)$ bpcu, which exceeds the SISO system spectral efficiency of $\log_2(M)$ bpcu by a factor of N_T.

This benefit of the SMUX-MIMO comes with a requirement of a careful approach for mitigating the effect of MIMO channel matrix $\boldsymbol{H}$ [3]. Certain assumptions can be made about the MIMO channel matrix, here we have assumed to have the perfect CSI available to us. Practically this CSI can fairly be obtained by sending pilot symbols from transmitter to receiver for channel estimation. The CSI obtained at the receiver is termed as channel state information at the receiver (CSIR). One way to obtain the CSI at the transmitter is to provide a feedback path between transmitter and receiver so that CSI obtained at the receiver can be sent back to the transmitter. CSIT is more preferable then CSIR. The drawback of CSIR is the increased complexity of receiver [12] at the mobile user because the mobile handset is typically smaller. The base station can handle the complexity as the size is quite larger. The main benefit CSIT provides is the reduction of complexity of the receiver.

At the MIMO receiver, different types of receivers can be used for the successful decoding of the received signal. ML receiver is the most optimal receiver. In ML receiver, we take the received signal vector and subtract it from all the possible combinations of the transmitted vector and then we select the vector which gives us the minimum norm square as the transmitted vector and we finally get the estimated information bearing vector. The mathematics for ML receiver is shown in the below equation

$$\hat{\boldsymbol{x}} = \arg\min_{X} \|\boldsymbol{y} - \boldsymbol{H}\boldsymbol{X}\|^2 \tag{2.3}$$

$\hat{\boldsymbol{x}}$ is the estimated information-bearing vector and $\boldsymbol{X}$ contains all possible outcomes for the transmit vector. ML receiver has very high computational complexity and it increases exponentially with an increase in the constellation size. Apart from ML receiver, there is ZF receiver which as also be used to for the estimation of the transmit information-bearing vector. The mathematics for the zero-forcing receiver is shown below

$$\hat{\boldsymbol{x}} = (\boldsymbol{H}^H\boldsymbol{H})^{-1}\boldsymbol{H}^H\boldsymbol{y}, \tag{2.4}$$

$$\hat{\boldsymbol{x}} = (\boldsymbol{H}^H\boldsymbol{H})^{-1}\boldsymbol{H}^H(\boldsymbol{H}\boldsymbol{x} + \boldsymbol{n}), \tag{2.5}$$

Since, $(\boldsymbol{H}^H\boldsymbol{H})^{-1}\boldsymbol{H}^H\boldsymbol{H} = \boldsymbol{I}$ matrix. Therefore, the effect of $\boldsymbol{H}$ matrix is removed and We get

$$\hat{\boldsymbol{x}} = \boldsymbol{x} + (\boldsymbol{H}^H\boldsymbol{H})^{-1}\boldsymbol{H}^H\boldsymbol{n} \tag{2.6}$$

Here $\hat{\boldsymbol{x}}$ is estimated information-bearing vector with noise. We can use decision boundary based decoding for the estimation of the transmitted symbols. In ZF receiver, there is possibility of noise amplification which is not good for the receiver. There are other MIMO receivers as well like Sphere Decoding, and Minimum Mean Square Error (MMSE) receivers etc. Different receivers have different qualities. The complexity and the error performance of a receiver can be traded-off.

2.2 Zero-Forcing (ZF) Precoded MIMO

As we saw that ZF MIMO receiver suffers from noise amplification. Also, using CSI at the receiver further increases the complexity of the receiver. The use of CSI at the transmitter reduces the receiver complexity. We can use CSIT by performing transmit precoding. ZF precoding technique can be used at the transmitter. ZF technique inverts the MIMO channel matrix. ZF precoding matrix is denoted by $\boldsymbol{P}$ matrix. The role of this $\boldsymbol{P}$ matrix is to perform the pseudo inverse of the MIMO channel matrix. This $\boldsymbol{P}$ matrix is of dimension N_T x N_R and is given by

$$\boldsymbol{P} = \boldsymbol{H}^H(\boldsymbol{H}\boldsymbol{H}^H)^{-1}, \tag{2.7}$$

where $\boldsymbol{H} \in \mathbb{C}^{N_R \times N_T}$ denotes the MIMO channel matrix. Perfect knowledge of CSI is assumed at the transmitter. Therefore, the $\boldsymbol{H}$ matrix is perfectly known at the transmitter. The transmit vector is denoted by $\boldsymbol{s}$. The information-bearing vector is N_R dimensional with each element belonging to M-ary APM constellations. The information-bearing vector is precoded with the zero-forcing precoder at the transmitter. Now, the final precoded transmit vector $\boldsymbol{s}$ becomes

$$\boldsymbol{s} = \beta \boldsymbol{P}\boldsymbol{x}, \tag{2.8}$$

where β is the power normalizing factor ensuring that $\mathbb{E}[||\boldsymbol{s}||^2] = 1$. The formula for the power normalizing factor (β) is given by

$$\beta = \sqrt{\frac{1}{Tr(\boldsymbol{H}\boldsymbol{H}^H)^{-1}}} \tag{2.9}$$

The received signal $\boldsymbol{y}$ for the ZF precoded MIMO is given by

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{s} + \boldsymbol{n}, \tag{2.10}$$

where $\boldsymbol{n} \sim \mathcal{CN}(0, \sigma^2\boldsymbol{I})$ is the Additive White Gaussian Noise (AWGN) with zero mean and variance $\sigma^2\boldsymbol{I}$ at the receiver. Now, since $\boldsymbol{s} = \boldsymbol{P}\boldsymbol{x}$ by substituting $\boldsymbol{s}$ in above equation we get,

$$\boldsymbol{y} = \beta \boldsymbol{H}\boldsymbol{P}\boldsymbol{x} + \boldsymbol{n}, \tag{2.11}$$

We know that $\boldsymbol{P} = \boldsymbol{H}^H(\boldsymbol{H}\boldsymbol{H}^H)^{-1}$. By substituting the value of $\boldsymbol{P}$ in the above equation we get,

$$\boldsymbol{y} = \beta(\boldsymbol{H}\boldsymbol{H}^H)(\boldsymbol{H}\boldsymbol{H}^H)^{-1}\boldsymbol{x} + \boldsymbol{n}, \tag{2.12}$$

Now, since $(\boldsymbol{H}\boldsymbol{H}^H)(\boldsymbol{H}\boldsymbol{H}^H)^{-1}$ = $\boldsymbol{I}$ matrix. Therefore, received signal vector $\boldsymbol{y}$ becomes

$$\boldsymbol{y} = \beta \boldsymbol{x} + \boldsymbol{n}, \tag{2.13}$$

which is same as AWGN channel. By using the ZF precoding technique, the problem of noise amplification is removed. Also, the complexity of the receiver is reduced. The spectral efficiency of zero-forcing precoded MIMO is given by

$$SE_{ZF-MIMO} = N_R \times \log_2(M) \tag{2.14}$$

2.3 SM-MIMO

Despite having numerous benefits, the MIMO system does have a few practical issues like - Inter Channel Interference (ICI), Transmit Antenna Synchronization (TAS), and requirement of RF chains [5, 13]. Each transmit antenna requires a dedicated RF Hardware Chain, thereby increasing the system cost. The energy efficiency decreases with an increase in the number of antennas (RF chains) as it mainly depends on the power amplifier. Also, not to forget the increased complexity of the MIMO system.

The solution to the above-mentioned issues is Spatial Modulation MIMO (SM-MIMO) system. Spatial Modulation is a technique where synchronization is not required between the antennas at the transmitter and completely negates ICI. In the SM-MIMO, only one of the N_T transmit antennas is activated over a symbol duration. In addition to using the M-ary APM constellations, SM-MIMO also transmits information using the transmit antennas indexes. [5]. The spectral efficiency of SM-MIMO is given by

$$SE_{SM} = \log_2(N_T) + \log_2(M) \tag{2.15}$$

For SM-MIMO, the transmit vector is an N_T dimensional vector given by

$$\mathbf{s} = \mathbf{x} = [0, 0, ..., x_t, ...0]^T \tag{2.16}$$

The received signal vector for SM-MIMO is given below

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}, \tag{2.17}$$

The above equation simplifies to

$$\mathbf{y} = \mathbf{h_t} x_t + \mathbf{n}, \tag{2.18}$$

where $\mathbf{h_t}$ is the t-th column of H and x_t is the APM symbol transmitted via the t-th transmit antenna.

At the receiver, we can use the ML decoder for the estimation of the activated transmit antenna and transmitted APM symbol. The drawback of the SM-MIMO system is its spectral efficiency. SM-MIMO has a spectral efficiency of $\log_2(N_T\, M)$ bpcu, which is greater than the SISO system's spectral efficiency but smaller than the SMUX-MIMO system. Also, the SM-MIMO system requires the use of CSIR, which eventually increases the receiver complexity. Therefore, increasing the implementation cost at the receiver.

2.4 PSM-MIMO

An alternative to the SM-MIMO that simplifies the receiver design is the transmit-precoding-aided SM-MIMO (or PSM-MIMO in short). PSM-MIMO is simply the spatial modulation at the receiver end. Therefore, PSM-MIMO is also known as Receiver spatial modulation (RSM). PSM-MIMO uses the CSI at the transmitter. [14]. In the PSM-MIMO, the additional information is transmitted in the spatial dimension by selecting one of N_R receive antennas [7, 15–17]. The spectral efficiency of PSM-MIMO is given by

$$SE_{PSM} = \log_2(N_R) + \log_2(M) \tag{2.19}$$

The technique of ZF transmit precoding is employed with CSIT in the PSM-MIMO system. For PSM-MIMO, the information bearing vector ($\boldsymbol{x}$) is N_R dimensional and given by

$$\boldsymbol{x} = [0, 0, ..., x_r, ...0]^T \tag{2.20}$$

The transmit vector ($\boldsymbol{s}$) is N_T dimensional and precoded version of the information-bearing vector. The transmit vector ($\boldsymbol{s}$) is given by

$$\mathbf{s} = \beta \mathbf{P} \mathbf{x} \tag{2.21}$$

where β is the power normalizing factor ensuring that $\mathbb{E}[||\boldsymbol{s}||^2] = 1$. The formula for the power normalizing factor (β) is given by

$$\beta = \sqrt{\frac{N_R}{Tr(\boldsymbol{H}\boldsymbol{H}^H)^{-1}}} \tag{2.22}$$

The received signal $\boldsymbol{y}$ for the ZF precoded MIMO is given by

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{s} + \boldsymbol{n}, \tag{2.23}$$

Now, since $\boldsymbol{s} = \boldsymbol{P}\boldsymbol{x}$ by putting $\boldsymbol{s}$ in above equation we get,

$$\boldsymbol{y} = \beta\boldsymbol{H}\boldsymbol{P}\boldsymbol{x} + \boldsymbol{n}, \tag{2.24}$$

We know that $\boldsymbol{HP} = \boldsymbol{I}$ matrix. By substituting the value of $\boldsymbol{HP}$ in the above equation, the received signal vector $\boldsymbol{y}$ simplifies as

$$\boldsymbol{y} = \beta\boldsymbol{x} + \boldsymbol{n} \tag{2.25}$$

PSM-MIMO does remove one of the drawbacks of SM-MIMO, which is the receiver implementation cost but the only problem PSM-MIMO has is its spectral efficiency. The throughput/spectral efficiency of the PSM-MIMO system is $\log_2(N_R\, M)$ bpcu. As the number of antennas are generally smaller at the mobile station compared to the base station. The spectral efficiency of the PSM-MIMO system even reduces than the SM-MIMO system.

CHAPTER 3

Problem Statement

The APM spectral efficiency is $\log_2(M)$ bpcu. MIMO offers a multiplicative increase in the spectral efficiency compared to the APM spectral efficiency. The spectral efficiency of MIMO system is given by

$$SE_{MIMO} = min(N_T, N_R) \times \log_2(M) \tag{3.1}$$

The maximum realizable spectral efficiency of the SM-MIMO is

$$SE_{SM} = \log_2(N_T) + \log_2(M) \tag{3.2}$$

PSM-MIMO which offers reduction in the complexity of the SM-MIMO has the maximum realizable spectral efficiency of

$$SE_{SM} = \log_2(N_R) + \log_2(M) \tag{3.3}$$

The spectral efficiency of both SM-MIMO and PSM-MIMO exceeds the APM spectral efficiency of $\log_2(M)$ bpcu but the spectral efficiency of SM-MIMO and PSM-MIMO increases logarithmically. Despite having numerous advantages of SM-MIMO and PSM-MIMO, the drawback of both schemes is their spectral efficiency. The throughput (spectral efficiency) of SM-MIMO and PSM-MIMO is lower than the MIMO. Generally $N_T > N_R$, therefore the spectral efficiency of PSM-MIMO system is even less than SM system.

For example, consider a 8 x 8 MIMO system. To increase the spectral efficiency by 2 bits, we will require 24 more antennas at the receiver. At the mobile station, it is not possible to deploy 24 more antennas. Also, the associated cost would be high. Furthermore, every one-bit increment would require an exponential increase in the number of receieve antennas.

In this thesis, our aim is to increase the spectral efficiency of PSM-MIMO system as well as the zero-forcing (ZF) precoded MIMO system without increasing the number of receive antennas at the mobile station (receiver).

CHAPTER 4
Literture Survey

In regard to the PSM-MIMO system, several other schemes have been developed in the literature to increase the throughput/spectral efficiency of the PSM-MIMO system. The paper in [9] proposes a new technique of MIMO transmission that improves the spectral efficiency of the PSM-MIMO system . The concept for the proposed work comes from generalized spatial modulation (GSM), in which we activate a specific group of transmit antennas, and the additional information to be sent is carried by the pattern of the activated antenna. In the proposed Generalized PSM (GPSM), the idea is to activate a specific subset of the receive antenna and the activated receive antenna pattern will be carrying the information. Let subset of the activated receive antennas is denoted by Q. Q contains all the possible combinations of the activated receive antenna. Now, the spectral efficiency for the GPSM system is

$$SE_{GPSM} = \log_2(Q) + \log_2(M) \tag{4.1}$$

This paper provides both the analytical and numerical results for the proposed GPSM scheme. They have performed the low rank approximation for the large dimensional MIMO system. Simulation results of this paper shows that GPSM is a promising alternative way of MIMO-based communication with higher throughput compared to the PSM-MIMO system. Also, the proposed reinforcement matrix for the proposed GPSM provides a further improvement in performance.

A new technique that further increases the spectral efficiency of the GPSM-MIMO system is proposed [10]. This paper discusses the possibility of using quadrature spatial modulation (QSM) at the receiver end integrated with the GPSM MIMO scheme. The concept of the proposed GPQSM is that the SM works in both the in-phase as well as quadrature components of the received signals. Therefore, the proposed GPQSM can deliver more information bits than GPSM. This paper also derives the closed-form upper bound on the average bit error probability (ABEP) for proposed GPQSM.

A novel technique for the GPSM MIMO system is proposed that improves the spectral efficiency of PSM [18]. The proposed technique is receive antenna switching for GPSM system. Receive antenna switching refers to the change in the active receive antennas throughout a symbol period. GPSM via receive antenna switching (which can also be termed as transition) has the capability to further enhance the error performance of the GPSM system. In GPSM via receive antenna switching, the transmit precoding switches once at the halfway point of the symbol duration, transferring one group of the active receive antennas to another group of active receive antennas. As a result of this switching, the transition pattern of the receive antennas is now capable of transmitting more information bits. The proposed GPSM via receive antenna switching has two advantages. First, the error performance increases significantly and secondly, GPSM via receive antenna transition requires less number of receive antennas compared to the PSM-MIMO and GPSM-MIMO system with same spectral efficiency. This paper increases the spectral efficiency of the PSM system using the concept of GPSM with receive antenna transition.

A new transmission scheme is proposed in [19]. They are proposing to partition the transmit antennas into N_t transmit antenna group (TAG) and receiver antennas into N_r receive antenna group (RAG). They are using the idea of the SM at the transmitter end and receiver end. The idea of GSM has also been expanded to a new precoding-aided massive MIMO system, namely in an activated antenna group at both the transmitter and receiver ends. The proposed scheme is named as an enhanced receive GSM (ERGSM) system. For the proposed ERGSM method, they have also suggested a low complexity sub-optimal detection technique. Additionally, antenna grouping at the transmitter and receiver enhances the system's performance and boosts throughput.

A new power allocation technique has been explored in [17]. The proposed work explores the advantages of power allocation for the PSM-MIMO system. PSM-MIMO has been studied with both a per-antenna power limitation (PAPC) and a total transmit power constraint (TTPC). To determine power distribution parameters for the numerous receive antennas, they have suggested a simple iterative technique based on their derived solution and error vector reduction method. They have also considered the more practical PAPC power allocation for PSM-MIMO system. They have proposed an approximate convex optimization (ACO)-based iterative algorithm for power allocation for the latter case. According to their simulation findings, the proposed (EVR and ACO) power allocation methods outperform equal-power-allocated PSM- and power allocation based spatial multiplexing systems in terms of BER performance.

A new PSM-MIMO technique is proposed in [20].The proposed scheme is capable of achieving transmit and receive diversity (TRD) and therefore, the proposed scheme is named as TRD-PSM-MIMO scheme. In the TRD-PSM-MIMO system, information is sent jointly using the indices of the activated receive antenna and APM modulation. At the transmitter, they have used two types of different precoders referred to as ZF and MMSE. Utilizing these precoders has the advantage of making the receiver less complex. At the receiver, they have developed many detection algorithms, including the joint maximum likelihood detector (JMLD), a simplified JMLD, a ratio-threshold-test-assisted maximum likelihood detector (RTT-MLD), a successive maximum likelihood detector (SMLD), and a simplified SMLD. Additionally, they have analyzed the ABEP for the TRD-PSM-MIMO system with ZF and MMSE precoding. Based on the simulation findings among the proposed detectors, the JMLD has optimum BER performance at the expense of being the most complex. SMLD has the worst BER performance but it offers low complexity. RTT-MLD is capable of achieving an equivalent BER performance with a simplified JMLD. Simplified JMLD BER performance is close to JMLD. RTT-MLD has quite lower complexity compared to simplified JMLD.

The paper in [21] proposes a method to improve the SER performance of the PSM. The proposed method names indices of receive antennas as the spatial constellation in contrast to signalling constellation. The proposed work focuses on making SER better in order to achieve a good balance between the spatial and the signaling constellation. The proposed work is named as signalling-spatial constellation trade-off. They take the size of the signaling constellation to be four, which accomplishes a considerable trade-off while limiting the SER upper bound for PSK modulation. i.e., the QPSK. Their numerical findings demonstrate that, for the same spectral efficiency, the signalling-spatial constellation trade-off established by the obtained SER upper bound outperforms previous PSM-MIMO methods.

The paper in [22] proposes a new precoder for PSM-MIMO. It explores the use of minimum mean square error (MMSE) precoding for the PSM-MIMO system. They have derived the upper bound for the ABEP for MMSE precoded PSM-MIMO system. Simulation findings shows that the bound is tight for the high SNR values. They have also analyzed the impact on the ABEP of varying the number of transmit and receive antennas. It is evident that on increasing the number of transmit antennas the ABEP significantly improves. It is demonstrated that the MMSE precoded PSM-MIMO system outperforms the ZF precoded PSM-MIMO system in terms of performance.

Since the development of the PSM-MIMO system there have been continuous improvements and addition to the PSM-MIMO system. This paper discusses another addition to the PSM-MIMO system [23] where a class of efficient receive antenna selection (ERAS) is proposed. This paper aims to find the best receive antenna choice for the PSM system. They have analyzed at the RAS-based PSM system's upper bound performance in two separate methods. Firstly, by employing the condition number and eigenvalue, and secondly, by leveraging the channel matrix's Wishart distribution feature. Their simulation findings shows that compared to traditional RAS-PSM systems, the suggested ERAS technique is capable of greatly improving BER performance. The proposed WD-ERAS system is nearly capable of providing near-optimal performance while also requiring less computational effort.

The paper in [24] proposes two new detection algorithms with reduced complexity for the GPSM system. First algorithm is based on lattice reduction-zero forcing and the second is based on lattice reduction-MMSE. According to their simulation findings, each of the suggested methods can provide BER performance that is nearly as good as ML detection, but with a lot less complexity.

The paper in [15] proposes an optimal joint transmit and receive antenna (JTRASs) subsets choosing method for the PSM-MIMO system. JTRASs selection is done by the exhaustive search and it has a very high complexity. Also, it is tough to analyze the diversity gain because of its exhaustive search method. To deal with this, authors have proposed to use a decoupled transmit and receive antenna choosing method. Decoupled transmit and receive antenna choosing method first chooses receive antenna subset (RAS) and then selects transmit antenna subset (TAS). If we decrease the number of active transmit antennas by TAS choice after RAS choice it is analyzed that it will always make the BER performance poorer. The simulation results verify the analysis results. Their simulation and analytical findings can be seen as expansions of the work already done on RAS and TAS for PSM-MIMO systems. Furthermore, authors have proposed and compared two algorithms for transmit and receive antenna subset (TRAS) selection. In the first suggestion, the algorithms used for separable RAS and TAS successive selection, respectively, are incremental and decremental. The first proposal has an excellent performance. In comparison to the joint optimal and decoupled optimal algorithms, the computational complexity of the first suggested decoupled suboptimal TRAS selection technique is much lower. Second, the decremental technique in the first suggested TRAS selection algorithm is replaced with an incremental TAS selection approach, which significantly reduces computational complexity.

CHAPTER 5
Proposed Solution

In this chapter, two proposals have been proposed to increase the spectral efficiency of the existing PSM-MIMO system and MIMO system. Proposal one focuses on increasing the spectral efficiency of the PSM-MIMO system whereas proposal two focuses on the increase of spectral efficiency of the existing full-fledged MIMO system without increasing the number of transmit antenna or number of receive antenna. Our PoC is based on the ML detection at the receiver.

5.1 Proposal One

Proposal one presents a novel SMP-MIMO system technique. Proposal one aims to increase spectral efficiency beyond the existing spectral efficiency of the PSM-MIMO system without increasing the number of received antennas in the given PSM-MIMO system. Proposal one system comprises of bits splitter, virtual column vector selector and a two-stage precoding system at the transmitter. The diagram for the proposal one scheme is shown in Figure 5.1.

The first stage of precoding is a matrix precoder. This matrix precoder contains a matrix of horizontal type which means that the number of columns are much more than the number rows in the matrix. We refer this matrix precoder as an $\boldsymbol{A}$ matrix. This matrix precoder could be sparse matrix or non-sparse matrix. By the virtue of this $\boldsymbol{A}$ matrix, the spectral efficiency of proposal one system increases beyond the existing spectral efficiency of PSM-MIMO system without increasing the number of receive antennas or size of the M-ary constellation.

Now, if we use a non-sparse matrix as an $\boldsymbol{A}$ matrix we can increase the spectral efficiency but the SER performance of the system will be very poor. So, we have to be very careful while designing this $\boldsymbol{A}$ matrix. We need to design this $\boldsymbol{A}$ matrix keeping in mind that we need to reduce the interference among the elements of the information-bearing vector. On the contrary, if we use a sparse matrix as the matrix precoder. We can successfully design this $\boldsymbol{A}$ matrix to reduce the inter-

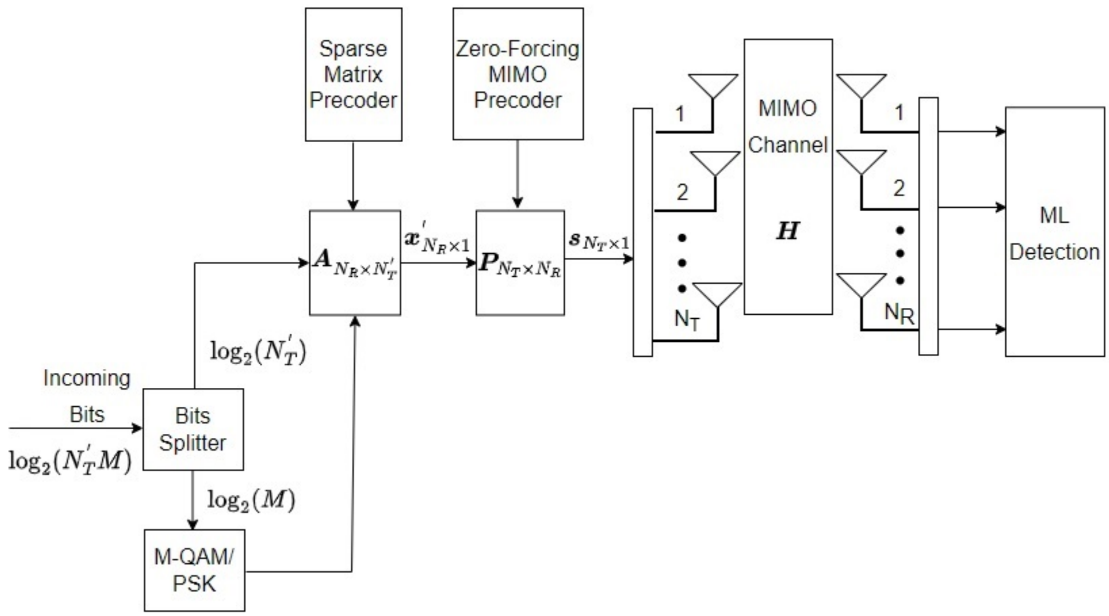

Figure 5.1: SMP-PSM-MIMO system model.

ference among the elements of the information-bearing vector as well as we can further leverage the sparseness of this $\boldsymbol{A}$ matrix to further reduce the computational complexity of the receiver.

The second stage of precoding is ZF precoder. We know that the transmitted signal is affected by the channel and we represent the effect of the channel by $\boldsymbol{H}$ matrix which is the MIMO channel matrix. This effect of the MIMO channel matrix is either countered at the receiver or at the transmitter. Since removing the effect of the MIMO channel matrix at the receiver increases the receiver complexity. Now, to reduce the receiver complexity we use precoding at the transmitter which helps us removes the effect of the MIMO channel matrix. There are different precoders used for this objective but the most optimum precoder is the ZF precoder. We assume that the transmitter has the perfect knowledge of CSI. ZF performs the psuedo inverse of MIMO channel matrix [25]. ZF is also known as null-steering because it nullifies the multiuser interference in a MIMO system [3].

5.1.1 System Model Proposal One

Transmission

Take into consideration a MIMO system that consists of N_R receive antennas and N_T transmit antennas. Proposal one is combined of bits splitter, and a two-stage precoding at the transmitter. Incoming information bits gets splitted into $\log_2(N_T^{'})$ bits and $\log_2(M)$ bits by the bits splitter. The first stage precoding is done by a sparse matrix precoder and the second-stage precoding is done by the ZF precoder. In SMP-MIMO proposal one, the information bearing vector $\boldsymbol{x}$ is a $N_T^{'}$ dimensional vector, with only one element belonging to M-ary APM constellation and the rest of all entries are zero in the vector. The non-zero element in the information-bearing vector virtually selects a column of the sparse matrix.

Instead of directly transmitting the information-bearing vector $\boldsymbol{x}$, we precode information bearing vector $\boldsymbol{x}$ in two stages. In the first stage, the information-bearing vector $\boldsymbol{x}$ is precoded with a sparse matrix. This sparse matrix is denoted as $\boldsymbol{A}$ matrix of dimension N_R x $N_T^{'}$ and $N_T^{'} >> N_R$ and N_T. In the second stage, the precoded vector $\boldsymbol{x}^{'} = \boldsymbol{A}\boldsymbol{x}$ is again precoded with ZF precoder ($\boldsymbol{P}$ matrix). As we have already seen the role of this $\boldsymbol{P}$ matrix is to perform the pseudo inverse of the MIMO channel matrix. We assume that we have the perfect channel state information. Therefore, the $\boldsymbol{H}$ matrix is perfectly known at the transmitter. The transmit vector is denoted by $\boldsymbol{s}$ vector. Now, the final precoded transmit vector $\boldsymbol{s}$ becomes

$$\boldsymbol{s} = \beta \boldsymbol{P}\boldsymbol{A}\boldsymbol{x}, \tag{5.1}$$

where β is the power normalizing factor ensuring that $\mathbb{E}[||\boldsymbol{s}||^2] = 1$. The formula for the power normalizing factor (β) is given by

$$\beta = \sqrt{\frac{1}{Tr(\boldsymbol{A}^H((\boldsymbol{H}\boldsymbol{H}^H)^{-1})^H \boldsymbol{A})}} \tag{5.2}$$

Normalization of the power is required so that we can compare the proposal one to other existing MIMO systems and see how our proposed system is performing with respect to others under the same or unity power constraint.

Power Normalization Factor:

For the SMP-PSM-MIMO system, we have the transmit vector $\boldsymbol{s} = \beta\boldsymbol{P}\boldsymbol{A}\boldsymbol{x}$. To simplify our calculation, Let $\boldsymbol{R} = \boldsymbol{H}\boldsymbol{H}^H$ and $\boldsymbol{B} = \boldsymbol{A}^H(\boldsymbol{R}^{-1})^H\boldsymbol{A}$. To obtain the power normalization factor, We need to perform the norm square of our transmit vector $\boldsymbol{s}$, i.e., $||\boldsymbol{s}||^2 = \boldsymbol{s}^H\boldsymbol{s}$.

$$\boldsymbol{s}^H\boldsymbol{s} = \beta^2\boldsymbol{x}^H\boldsymbol{A}^H\boldsymbol{P}^H\boldsymbol{P}\boldsymbol{A}\boldsymbol{x} \tag{5.3}$$

For the calculation of $\boldsymbol{s}^H\boldsymbol{s}$, let us first find out the term $\boldsymbol{P}^H\boldsymbol{P}$. $\boldsymbol{P}^H\boldsymbol{P}$ helps us simplify the calculation of $\boldsymbol{s}^H\boldsymbol{s}$,

$$\boldsymbol{P}^H\boldsymbol{P} = [\boldsymbol{H}^H(\boldsymbol{H}\boldsymbol{H}^H)^{-1}]^H\boldsymbol{H}^H(\boldsymbol{H}\boldsymbol{H}^H)^{-1} \tag{5.4}$$

$$\boldsymbol{P}^H\boldsymbol{P} = [(\boldsymbol{H}\boldsymbol{H}^H)^{-1}]^H\boldsymbol{H}\boldsymbol{H}^H(\boldsymbol{H}\boldsymbol{H}^H)^{-1} \tag{5.5}$$

We assumed that $\boldsymbol{R} = \boldsymbol{H}\boldsymbol{H}^H$. so, the above equation can be written as

$$\boldsymbol{P}^H\boldsymbol{P} = (\boldsymbol{R}^{-1})^H\boldsymbol{R}\boldsymbol{R}^{-1} \tag{5.6}$$

Now, since $\boldsymbol{R}\boldsymbol{R}^{-1} = \boldsymbol{I}$ matrix. The above equation can be simplified as

$$\boldsymbol{P}^H\boldsymbol{P} = (\boldsymbol{R}^{-1})^H = [(\boldsymbol{H}\boldsymbol{H}^H)^{-1}]^H \tag{5.7}$$

Since, $\boldsymbol{P}^H\boldsymbol{P}$ = $(\boldsymbol{R}^{-1})^H$. Putting this in the above equation, we get

$$\boldsymbol{s}^H\boldsymbol{s} = \beta^2\boldsymbol{x}^H\boldsymbol{A}^H(\boldsymbol{R}^{-1})^H\boldsymbol{A}\boldsymbol{x} \tag{5.8}$$

We assumed that $\boldsymbol{B}$ = $\boldsymbol{A}^H$ $(\boldsymbol{R}^{-1})^H$ $\boldsymbol{A}$. By replacing $\boldsymbol{A}^H(\boldsymbol{R}^{-1})^H\boldsymbol{A}$ by $\boldsymbol{B}$. The above equation can be written as

$$\boldsymbol{s}^H\boldsymbol{s} = \boldsymbol{x}^H\boldsymbol{B}\boldsymbol{x} \tag{5.9}$$

Assuming $\mathbb{E}\left[|x_k|^2\right] = 1$ and $\mathbb{E}\left[x_k x_j^*\right] = 0$, for SMP-PSM-MIMO system, we get $\mathbb{E}\left[\mathbf{s}^H\mathbf{s}\right] = (\mathbf{B}) = Tr(\mathbf{A}^H(\mathbf{R}^{-1})^H\mathbf{A})$.

$$\mathbb{E}\left[\mathbf{s}^H\mathbf{s}\right] = (\mathbf{B}) = Tr(\mathbf{A}^H(\mathbf{R}^{-1})^H\mathbf{A}) \tag{5.10}$$

To achieve $\mathbb{E}\left[||s||^2\right] = 1$ i.e. $\mathbb{E}\left[\mathbf{s}^H\mathbf{s}\right] = 1$, we set

$$\beta = \sqrt{\frac{1}{Tr(\mathbf{A}^H(\mathbf{R}^{-1})^H\mathbf{A})}} \tag{5.11}$$

$$\beta = \sqrt{\frac{1}{Tr(\boldsymbol{A}^H((\boldsymbol{H}\boldsymbol{H}^H)^{-1})^H\ \boldsymbol{A})}} \tag{5.12}$$

At the receiver, the received signal which is denoted by $\boldsymbol{y}$ is given by

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{s} + \boldsymbol{n}, \tag{5.13}$$

Now, since $\boldsymbol{s} = \boldsymbol{P}\boldsymbol{A}\boldsymbol{x}$ by putting $\boldsymbol{s}$ in above equation we get,

$$\boldsymbol{y} = \beta\boldsymbol{H}\boldsymbol{P}\boldsymbol{A}\boldsymbol{x} + \boldsymbol{n}, \tag{5.14}$$

We know that $\boldsymbol{P} = \boldsymbol{H}^H(\boldsymbol{H}\boldsymbol{H}^H)^{-1}$. By substituting the value of $\boldsymbol{P}$ in the above equation we get,

$$\boldsymbol{y} = \beta(\boldsymbol{H}\boldsymbol{H}^H)(\boldsymbol{H}\boldsymbol{H}^H)^{-1}\boldsymbol{A}\boldsymbol{x} + \boldsymbol{n}, \tag{5.15}$$

Now, since $(\boldsymbol{H}\boldsymbol{H}^H)(\boldsymbol{H}\boldsymbol{H}^H)^{-1}$ = $\boldsymbol{I}$ matrix. Therefore, received signal vector $\boldsymbol{y}$ becomes

$$\boldsymbol{y} = \beta\boldsymbol{A}\boldsymbol{x} + \boldsymbol{n}, \tag{5.16}$$

Since information bearing vector $\boldsymbol{x}$ has only one non-zero element. It virtually selects one of the column of the $\boldsymbol{A}$ matrix and the further simplified expression can be written as

$$\boldsymbol{y} = \boldsymbol{a_t}x_l + n \tag{5.17}$$

where $\boldsymbol{a_t}$ is the t-th column of $\boldsymbol{A}$ matrix and x_l is the non-zero APM symbol. The above equation is similiar to AWGN channel.
Different $\boldsymbol{A}$ matrices used for proposal one are shown below

$$\boldsymbol{A}_{4\text{x}8} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \tag{5.18}$$

$$\boldsymbol{A}_{4\text{x}8} = \begin{bmatrix} 1 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{3}} & 0 \\ 0 & 1 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ 0 & 0 & 1 & 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ 0 & 0 & 0 & 1 & 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{3}} \end{bmatrix} \tag{5.19}$$

$$\boldsymbol{A}_{\text{LDPC}} = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \end{bmatrix} \tag{5.20}$$

The spectral efficiency of PSM-MIMO system is $\log_2(N_R)$ + $\log_2(M)$ bpcu. Now, since $N_T^{'} >> N_R$ and N_T. The Spectral efficiency for the SMP-PSM-MIMO system will increase beyond the existing PSM-MIMO system without any increase in the number of receive antenna or increase in the size of the M-ary constellation. Spectral efficiency for the SMP-PSM-MIMO system is given by the following expression

$$SE_{\text{SMP-PSM}} = \log_2(N_T^{'}) + \log_2(M)bpcu \tag{5.21}$$

Detection at the Receiver:

The objective is to find the transmitted APM symbol and the selected column of the $\boldsymbol{A}$ matrix given the received vector $\boldsymbol{y}$. To achieve our objective, we perform the Maximum Likelihood decoding at the receiver. In Maximum Likelihood decoding, we take the received vector and subtract it from all the possible combination of the transmitted vector and then we select the vector which gives us the minimum norm square as the transmitted vector and we finally get the estimated column of the $\boldsymbol{A}$ matrix and the transmitted APM symbol.

$$[\hat{\mathbf{t}}, \hat{l}] = \underset{t=1...N_T^{'}, l=1...M}{\text{argmin}} \|\mathbf{y} - \beta \mathbf{a}_t x_l\|^2 \tag{5.22}$$

Table 5.1 shows the system parameters comparison for the above-mentioned schemes where P1 denotes proposal one and Table 5.2 shows the spectral efficiency comparison for the aforementioned schemes.

Parameters	**PSM-MIMO**	**Proposal One**
Information Bearing Vector ($\mathbf{x}$)	N_R x 1	$N_T^{'}$ x 1
Spectral Efficiency	$log_2(N_R M)$	P1 - $log_2(N_T^{'} M)$
Transmit Vector ($\mathbf{s}$)	$\beta_{PSM}\mathbf{Px}$	$\beta_{SMP}\mathbf{PAx}$
Power Normalizing Factor (β)	$\sqrt{\frac{N_R}{trace(\mathbf{HH}^H)^{-1}}}$	$\sqrt{\frac{1}{trace(\mathbf{A}^H(\mathbf{HH}^H)^{-1})^H\mathbf{A}}}$
Received Signal ($\mathbf{y}$)	$\beta_{PSM}\mathbf{x} + \mathbf{n}$	$\beta_{SMP}\mathbf{Ax} + \mathbf{n}$

Table 5.1: System Parameters Comparison For aforementioned Schemes.

M	**PSM-MIMO**	**Proposal One**
2	3 bpcu	4 bpcu
4	4 bpcu	5 bpcu
8	5 bpcu	6 bpcu
16	6 bpcu	7 bpcu
32	7 bpcu	8 bpcu
64	8 bpcu	9 bpcu
128	9 bpcu	10 bpcu
256	10 bpcu	11 bpcu

Table 5.2: Spectral Efficiency Comparison For aforementioned Schemes.

Simulation Results

For simulation, we have considered 4-QAM transmission with $N_T = 4$ and $N_R = 4$. We have shown the simulated results for SER performance of proposal one system and also the comparison of SER for PSM-MIMO and proposal one. We performed 10000 Monte Carlo simulations to obtain the following results.

Figure 5.2 shows the SER performance for proposal one with different $\boldsymbol{A}$ matrices. The magenta line shows the SER performance with $\boldsymbol{A}$ matrix having all elements as one. As we can see the SER performace for this is $\boldsymbol{A}$ matrix is very poor. The blue line shows the SER performance with $\boldsymbol{A}$ matrix having randomly choosen elements. We can clearly see the improvement in the SER performance for randomly choosen $\boldsymbol{A}$ matrix. To further improve the SER performance of proposal one, we carefully designed a sparse matrix. The designed sparse matrix

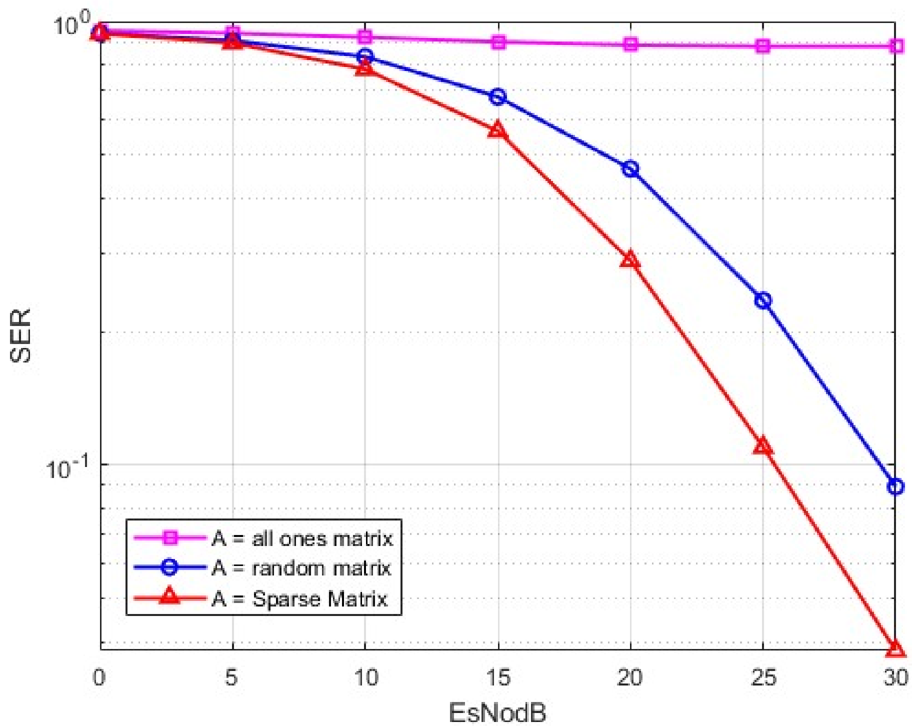

Figure 5.2: SMP-MIMO (proposal one) SER performance for different $\boldsymbol{A}$ matrices.

is shown in eq.(5.20) and the red line on shows the SER performance for the designed sparse $\boldsymbol{A}$ matrix. We can clearly see that it further improves the SER performance of proposal one.

Figure 5.3 shows the simulation result for the different errors possible for the proposed work. As we estimate the choosen column of the $\boldsymbol{A}$ matrix, if our estimated column is not correct then it will result in the column error which is shown by the red curve. Error in the detection of the transmitted APM symbol will result in symbol error which is shown by the blue curve in the plot. Now, as any one of the error occurs in the system it will result in an overall error for the proposed system which is said as the combined error for the proposed system. The combined error for the proposed system is shown as the black line in the plot. In Figure 5.3, since we used 4-QAM the column error is more dominating.

Figure 5.4 shows the simulation result for the different errors possible for the proposed work with 16-QAM. In this, we can clearly see that symbol error is dominating. As in QAM, probability of error increases with M.

Figure 5.5 shows the comparison of the spectral efficiency for the proposed system with PSM-MIMO system with $N_T = 4$, $N_R = 4$ and $N_T^{'} = 8$. The spectral efficiency of proposal one is $\log_2(N_T^{'}M)$ bpcu and for PSM-MIMO is $\log_2(N_R M)$ bpcu. The orange line shows the spectral efficiency curve for proposal one and

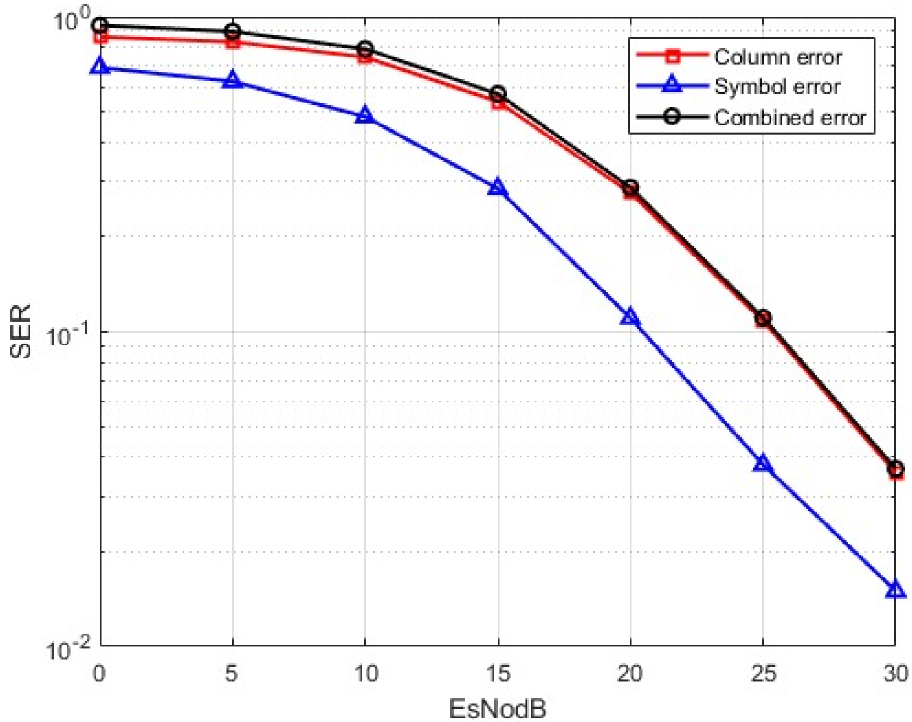

Figure 5.3: SMP-MIMO (proposal one) Column error, Symbol error and Combined error $M = 4$.

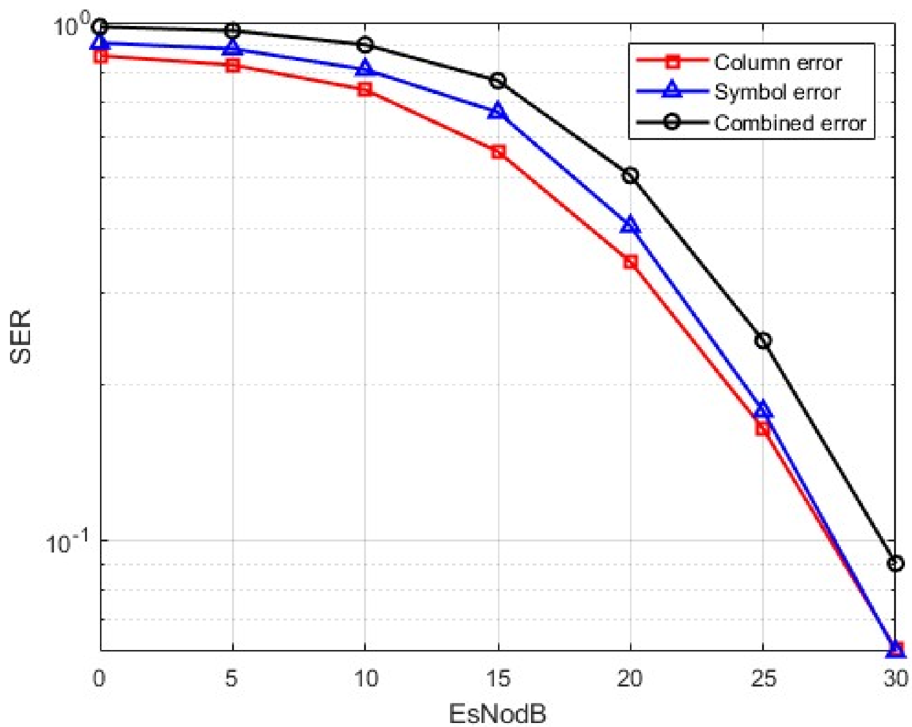

Figure 5.4: SMP-MIMO (proposal one) Column error, Symbol error and Combined error with $M = 16$.

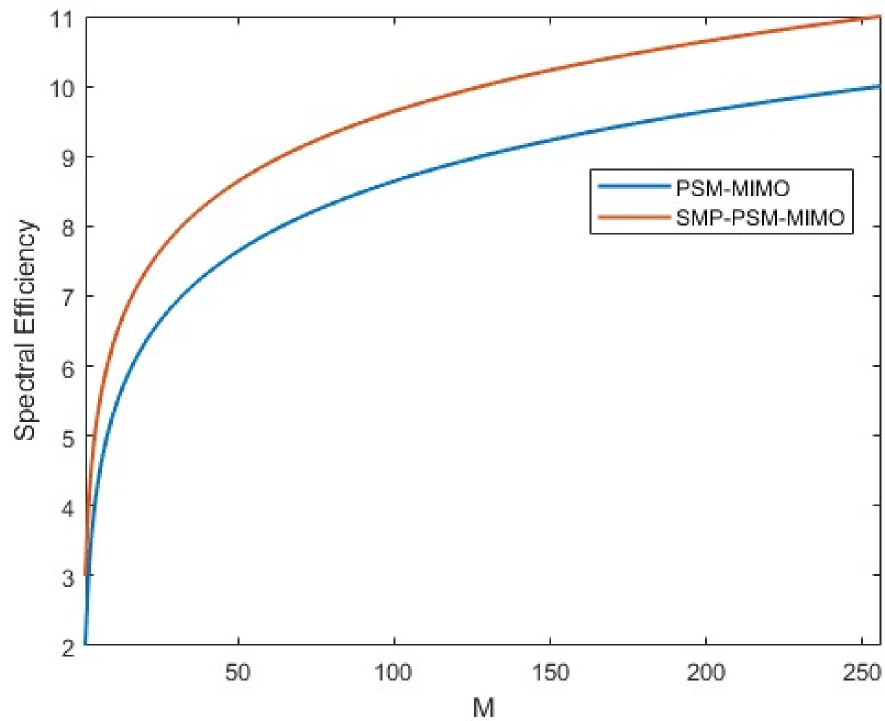

Figure 5.5: Spectral efficiency comparison of SMP-MIMO (proposal one) with PSM-MIMO.

the blue line shows the spectral efficiency curve for the PSM-MIMO system. From Figure 5.4, we can clearly see the significant increase in the spectral efficiency for proposal one.

Figure 5.6 shows the SER performance comparison between the proposed system and the PSM-MIMO system. The red curve shows the SER performance for proposal one system with system parameters ($N_T = 4$, $N_R = 4$, $M = 4$). The black curve shows the SER performance for the PSM-MIMO system with same system parameters. We can clearly see that the SER performance of the PSM-MIMO system is still better than the proposed system. Since the proposed system is a full-fledged MIMO system we are able to increase the spectral efficiency beyond the PSM-MIMO system but the cost of degraded SER performance.

As we increased the number of the transmit antenna by one, we can see a significant increase in the SER performance for both systems because of the increase in the the value of β. The blue curve shows the SER performance for proposal one with system parameters ($N_T = 5$, $N_R = 4$, $M = 4$). The magenta curve shows the SER performance for the PSM-MIMO system with same system parameters. The study for the exact improvement of 5×4 over the 4×4 system is still pending. The SER performance improves for the proposed system but the PSM-MIMO sys-

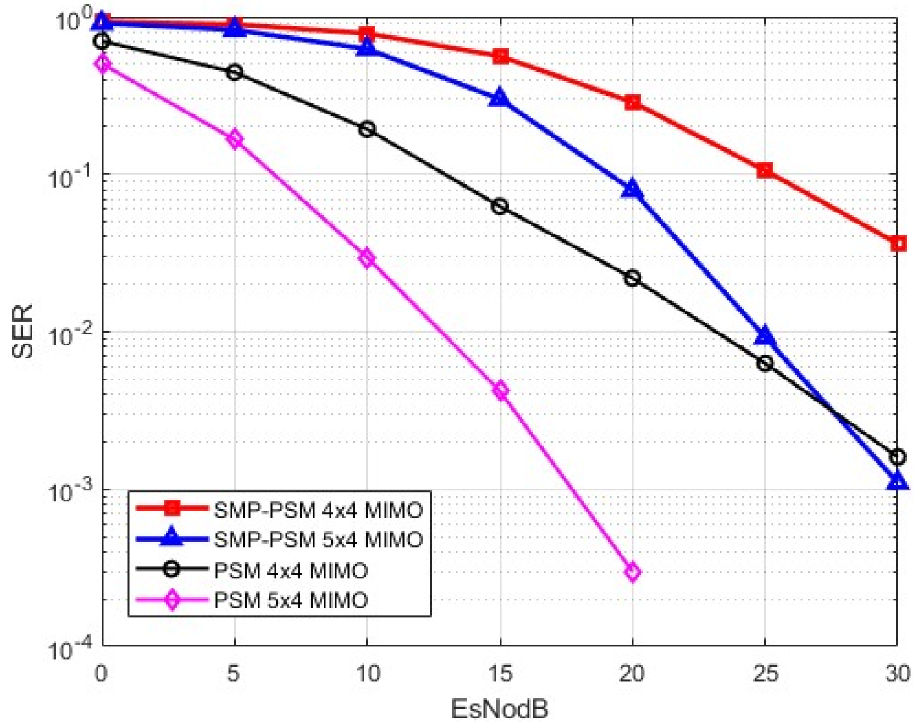

Figure 5.6: SER comparison of SMP-MIMO (proposal one) with PSM-MIMO.

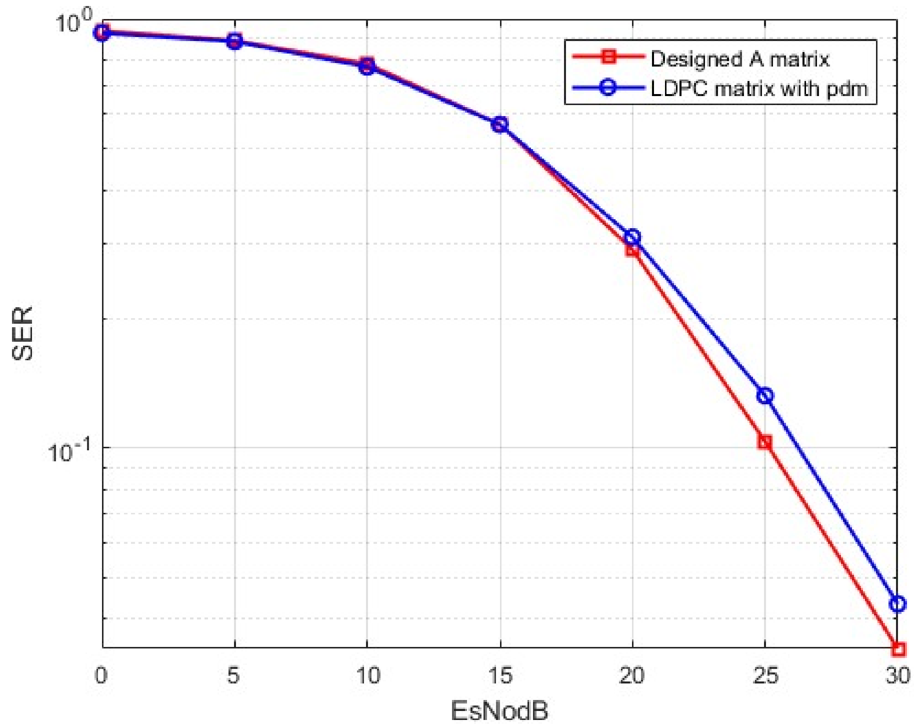

Figure 5.7: Comparison of designed $\boldsymbol{A}$ matrix with LDPC matrix multiplied with a power domain multiplier for SMP-MIMO (proposal one).

tem still performs better than the proposed system but at less spectral efficiency.

Figure 5.7 shows the SER performance comparison for the proposed system with three different sparse matrices. LDPC matrix of dimension 4×8 has few columns are similiar to each other so when decoding the column index at the receiver it will give error. To eradicate this problem, we use an LDPC matrix multiplied with a power domain multiplier (pdm). The blue curve shows the performance of SMP-MIMO (proposal one) with LDPC matrix using a power domain multiplier. The performance of SMP-MIMO (proposal one) is almost similar to the designed $\boldsymbol{A}$ matrix. The red curve depicts the SER performance for the proposed system with the designed sparse $\boldsymbol{A}$ matrix shown in eq.(5.20). We used the LDPC matrix because of its sparseness but with Maximum Likelihood decoding it is not performing well. We can clearly observe that the designed sparse matrix outperforms the LDPC matrix. The LDPC matrix used for this simulation is shown in eq.(5.20).

5.2 Proposal Two

In proposal one, we are choosing one of the column of the sparse $\boldsymbol{A}$ matrix with the help of the information bearing vector with only one non-zero element. so, in proposal one the spectral efficiency is increasing logarithmically. Also, the proposal one is a full fledged MIMO system which means all the antennas are transmitting and receiving. So, the idea is to make the full use of MIMO system which means instead of virtually selecting one of the column of the $\boldsymbol{A}$ matrix use the complete $\boldsymbol{A}$ matrix. This will help the spectral efficiency increase in a linear manner.

Proposal two presents a novel technique of SMP-MIMO system. The SMP-MIMO system aims at increasing the spectral efficiency beyond the existing spectral efficiency of MIMO system without increasing the number of transmit antennas or received antennas in the given MIMO system. SMP-MIMO system comprises of two-stage precoding at the transmitter. The block diagram for the SMP-MIMO system is shown in Figure 5.8.

In SMP-MIMO, the first stage of precoding is the sparse matrix precoder ($\boldsymbol{A}$ matrix). We could use a non-sparse matrix precoder but the performance would be poor. Now, by virtue of this $\boldsymbol{A}$ matrix, we are increasing the spectral efficiency of the SMP-MIMO system beyond the spectral efficiency of the existing MIMO system without increasing the number of receive antennas or transmit antennas and size of the M-ary constellation.

As we know that we have to be very careful while designing this $\boldsymbol{A}$ matrix. We need to design this $\boldsymbol{A}$ matrix keeping in mind that we need to reduce the interference among the elements of the information bearing vector. In SMP-MIMO (proposal two) system, the information bearing vector is not same as proposal one system. In SMP-MIMO, each element of the information-bearing vector belongs to the M-ary APM constellation. We need to design this $\boldsymbol{A}$ matrix to reduce the interference among the elements of the information-bearing vector as well as to further leverage the sparseness of this $\boldsymbol{A}$ matrix to further reduce the computational complexity of the receiver.

In SMP-MIMO, the second stage of precoding is ZF precoder. We try to remove the effect of $\boldsymbol{H}$ matrix at the transmitter by performing precoding at the transmitter. This also helps in reducing the complexity of the receiver. Different precoders are used for this objective but the most optimum precoder is the ZF precoder. We assume that we have the perfect knowledge of CSI at the transmitter.

At the receiver, we need to decode the received symbol correctly. For that,

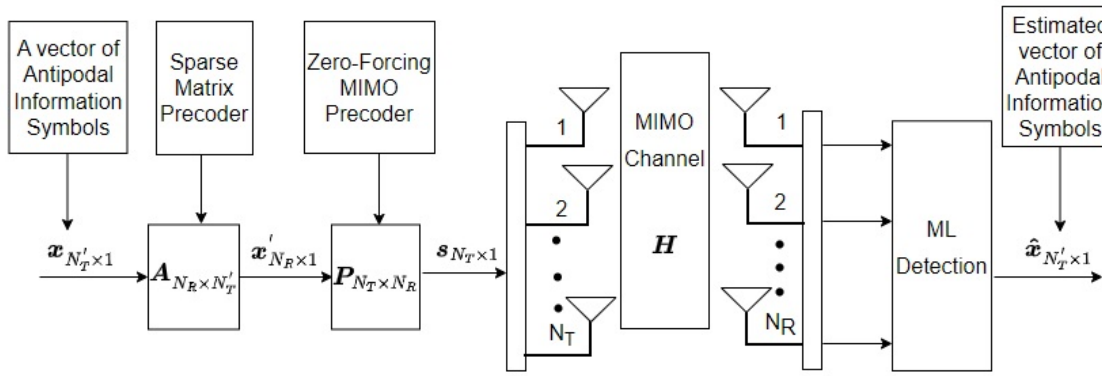

Figure 5.8: SMP-MIMO system model.

we are using ML decoder. We demonstrate proof of the SMP-MIMO concept by simulation-driven experiments. Our PoC is based on the ML detection at the receiver.

5.2.1 Proposed SMP-MIMO System Model

Transmission

Consider a MIMO system with N_R receive antennas and N_T transmit antennas. SMP-MIMO introduces two-stage precoding in the system. Information bearing vector $\boldsymbol{x}$ is a $N_T^{'}$ dimensional vector, with each element belonging to the M-ary APM constellation. Instead of directly transmitting the information-bearing vector $\boldsymbol{x}$. We precode information bearing vector $\boldsymbol{x}$ in two stages. In the first stage, the information-bearing vector $\boldsymbol{x}$ is precoded with $\boldsymbol{A}$ matrix of dimension N_R x $N_T^{'}$. This $\boldsymbol{A}$ matrix is a sparse matrix and $N_T^{'} >> N_R$ and N_T. In the second stage, the precoded vector $\boldsymbol{x}^{'}$ = $\boldsymbol{Ax}$ is again precoded with $\boldsymbol{P}$ (ZF) matrix of dimension N_T x N_R. Now, the final precoded transmit vector ($\boldsymbol{s}$) becomes

$$\boldsymbol{s} = \beta \boldsymbol{PAx}, \tag{5.23}$$

where β is the power normalizing factor ensuring that $\mathbb{E}[||\boldsymbol{s}||^2] = 1$. We have already shown the derivation for power normalizing factor in proposal one. The formula for the power normalizing factor (β) is given by

$$\beta = \sqrt{\frac{1}{Tr(\boldsymbol{A}^H((\boldsymbol{H}\boldsymbol{H}^H)^{-1})^H\,\boldsymbol{A})}} \tag{5.24}$$

The received signal $\boldsymbol{y}$ is given by

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{s} + \boldsymbol{n}, \tag{5.25}$$

Now, since $\boldsymbol{s} = \boldsymbol{P}\boldsymbol{A}\boldsymbol{x}$ by putting $\boldsymbol{s}$ in above equation we get,

$$\boldsymbol{y} = \beta\boldsymbol{H}\boldsymbol{P}\boldsymbol{A}\boldsymbol{x} + \boldsymbol{n} \tag{5.26}$$

Since, $\boldsymbol{P} = \boldsymbol{H}^H(\boldsymbol{H}\boldsymbol{H}^H)^{-1}$ and we have already seen $\boldsymbol{H}\boldsymbol{P} = \boldsymbol{I}$ matrix. The above equation simplifies to

$$\boldsymbol{y} = \beta\boldsymbol{A}\boldsymbol{x} + \boldsymbol{n}, \tag{5.27}$$

which is similar to the AWGN channel. The $\boldsymbol{A}$ matrices used for SMP-MIMO is

$$\boldsymbol{A}_{4\text{x}6} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \tag{5.28}$$

$$\boldsymbol{A}_{4\text{x}6} = \begin{bmatrix} 1 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & 1 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & 0 & 1 & 0 & \frac{1}{\sqrt{2}} \end{bmatrix} \tag{5.29}$$

$$\boldsymbol{A}_{4\text{x}8} = \begin{bmatrix} 1 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{3}} & 0 \\ 0 & 1 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ 0 & 0 & 1 & 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ 0 & 0 & 0 & 1 & 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{3}} \end{bmatrix} \tag{5.30}$$

The spectral efficiency of MIMO system is min(N_T, N_R) x $\log_2(M)$ bpcu. Now, since $N_T^{'} >> N_R$ and N_T. The Spectral efficiency for the SMP-MIMO system will increase compared to the existing spectral efficiency of the MIMO system without any increase in the number of transmit antenna or receive antenna or increase in size of M-ary APM constellation. Spectral efficiency for the SMP-MIMO system is given by the following expression

$$SE_{\text{SMP}} = N_T' \log_2(M) bpcu \tag{5.31}$$

Detection at the Receiver:

Our objective is to find the transmitted information-bearing vector given the received vector $\boldsymbol{y}$. To achieve our objective, we perform the ML decoding at the receiver. In ML decoding, we take the received vector and subtract it from all the possible combinations of the transmitted vector. Then, we select the vector which gives us the minimum norm square as the transmitted vector and we finally get the estimated information-bearing vector.

$$\hat{\boldsymbol{x}} = \arg\min_{X} \|\boldsymbol{y} - \beta \boldsymbol{A}\boldsymbol{X}\|^2 \tag{5.32}$$

Theoritical Analysis for SER of SMP MIMO System

Let us consider Bpsk transmission. Now, from eq.(5.27), the Received SNR for k-th receive antenna will be

$$\gamma_k = \frac{\mathbb{E}[\beta^2 |x_k|^2]}{\mathbb{E}[|n_k|^2]} = \frac{\mathbb{E}[\beta^2]}{\sigma_n^2} \tag{5.33}$$

since $\mathbb{E}[|x_k|^2] = 1$, as the APM constellation considered is of average unit energy. Average received SNR at the receiver will be

$$\bar{\gamma} = \frac{\gamma_1 + \gamma_2 + ... + \gamma_{NR}}{N_R} = \frac{N_R \gamma_k}{N_R} = \gamma_k \tag{5.34}$$

When we consider BPSK symbols for transmission. Our information-bearing vector $\boldsymbol{x}$ (which contains BPSK symbols) is precoded by $\boldsymbol{A}$ matrix, i.e., after stage 1 precoding. Bpsk symbols gets converted to pulse amplitude modulated (PAM) symbols. So, to get the expression for probability of error of the SMP-MIMO system we will need the expression of SER for PAM symbols and the expression for the probability of error of PAM symbols is given by

$$P_e = \frac{2(M-1)}{M} Q\left(\sqrt{\frac{6\gamma}{M^2 - 1}}\right) \tag{5.35}$$

where γ denotes the average received SNR for PAM symbol transmission. By looking at eq.(5.27) we can see that the power normalization factor for the SMP-MIMO system (β) is multiplied to received PAM symbols. Therefore, the received

SNR in SMP MIMO scenario will be $\gamma_k = \beta^2\gamma$. So, the final expression for SER of the SMP-MIMO system will be

$$P_e = \frac{2(M-1)}{M} Q\left(\sqrt{\frac{6\bar{\gamma}}{M^2-1}}\right) \tag{5.36}$$

Since $\bar{\gamma} = \gamma_k$, The preceding calculation can also be expressed as

$$P_e = \frac{2(M-1)}{M} Q\left(\beta\sqrt{\frac{6\gamma_k}{M^2-1}}\right) \tag{5.37}$$

As we know $\gamma_k = \beta^2\gamma$, The preceding calculation can also be expressed as

$$P_e = \frac{2(M-1)}{M} Q\left(\sqrt{\frac{6\beta^2\gamma}{M^2-1}}\right) \tag{5.38}$$

$$P_e(x) = \frac{2(M-1)}{M} Q\left(\sqrt{\frac{6x^2\gamma}{M^2-1}}\right) \tag{5.39}$$

SER expression for SMP-MIMO (proposal two) is given by

$$SER = \int_{\beta_{min}}^{\beta_{max}} P_e(x) f_\beta(x)\, dx$$

where $f_\beta(x)$ is the pdf of the beta (normalizing factor).

Table 5.3 shows the system parameters comparison for above discussed MIMO schemes where P1 denotes proposal one and P2 denotes proposal two.

Parameters	PSM-MIMO	ZF-MIMO	SMP-MIMO
x	N_R x 1	N_R x 1	$N_T^{'}$ x 1
Spectral Efficiency	$log_2(N_R M)$	$N_R log_2(M)$	P1 - $log_2(N_T^{'} M)$ P2 - $N_T^{'} log_2(M)$
s	$\beta \mathbf{Px}$	$\beta \mathbf{Px}$	$\beta \mathbf{PAx}$
β	$\sqrt{\frac{N_R}{trace(\mathbf{HH}^H)^{-1}}}$	$\sqrt{\frac{1}{trace(\mathbf{HH}^H)^{-1}}}$	$\sqrt{\frac{1}{trace(\mathbf{A}^H(\mathbf{HH}^H)^{-1})^H \mathbf{A}}}$
y	$\beta \mathbf{x} + \mathbf{n}$	$\beta \mathbf{x} + \mathbf{n}$	$\beta \mathbf{Ax} + \mathbf{n}$

Table 5.3: System Parameters Comparison For aforementioned Schemes.

Table 5.4 shows the spectral efficiency comparison for the aforementioned Schemes

M	PSM-MIMO	SMP-MIMO (P1)	ZF-MIMO	SMP-MIMO (P2)
2	3 bpcu	4 bpcu	4 bpcu	8 bpcu
4	4 bpcu	5 bpcu	8 bpcu	16 bpcu
8	5 bpcu	6 bpcu	12 bpcu	24 bpcu
16	6 bpcu	7 bpcu	16 bpcu	32 bpcu
32	7 bpcu	8 bpcu	20 bpcu	40 bpcu
64	8 bpcu	9 bpcu	24 bpcu	48 bpcu
128	9 bpcu	10 bpcu	28 bpcu	56 bpcu
256	10 bpcu	11 bpcu	32 bpcu	64 bpcu

Table 5.4: Spectral Efficiency Comparison For aforementioned Schemes with ($N_T = 4, N_R = 4, N_T^{'} = 8$).

Simulation Results:

For simulation, we have considered BPSK transmission with $N_T = 4$ and $N_R = 4$. We have shown the simulated results for SER and BER performance of SMP-MIMO system. We have also shown the BER performance comparison between the ZF-precoded MIMO and SMP-MIMO system. Also, the theoretical and simulation result for the SER of SMP-MIMO is shown and the result matches for a given $\boldsymbol{A}$ matrix. We performed 100000 Monte Carlo simulations to obtain the following results.

Figure 5.9 shows the BER performance for proposal two (SMP-MIMO) with different $\boldsymbol{A}$ matrices. The red line shows the SER performance with $\boldsymbol{A}$ matrix having all elements as one. As we can see, the BER performance for this is $\boldsymbol{A}$

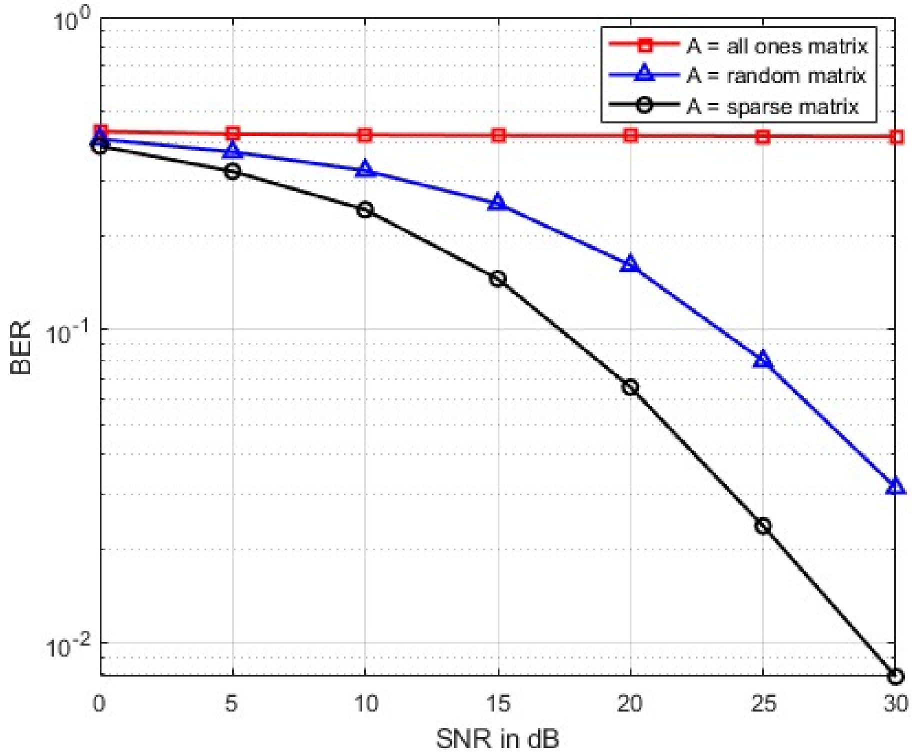

Figure 5.9: SMP-MIMO (proposal two) BER performance for different $\boldsymbol{A}$ matrices.

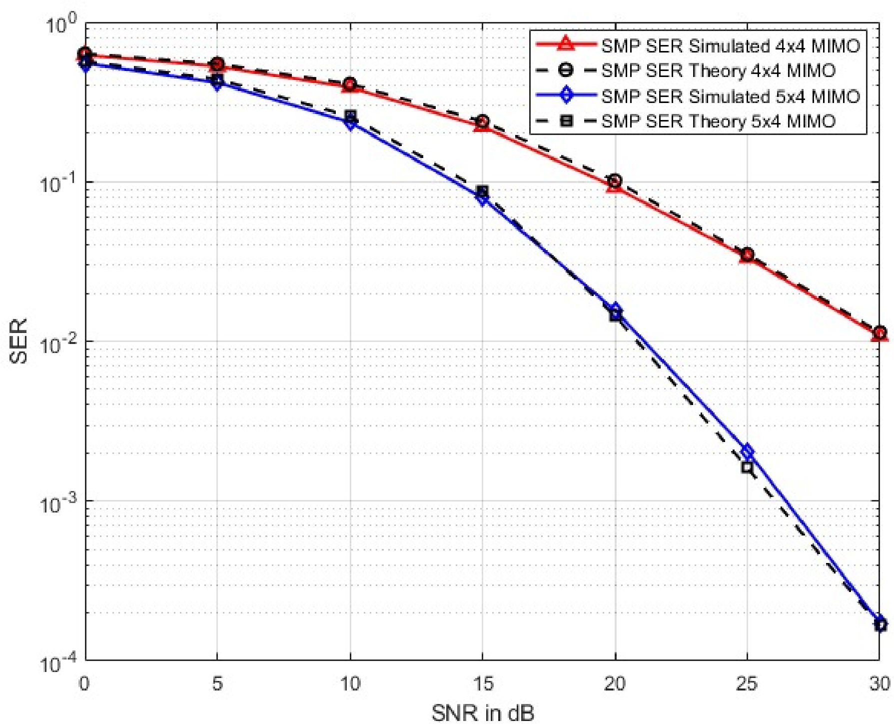

Figure 5.10: SMP-MIMO (proposal two) theoretical SER vs simulated SER performance.

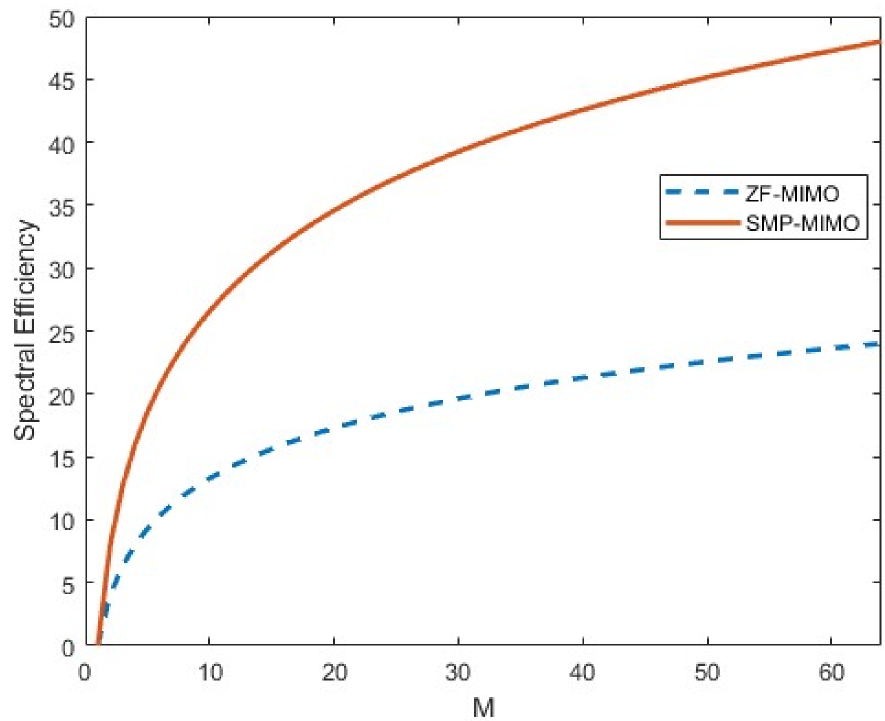

Figure 5.11: Spectral efficiency comparison of SMP-MIMO (proposal two) system with MIMO system.

matrix is very poor. The blue line shows the BER performance with $\boldsymbol{A}$ matrix having randomly chosen elements. We can clearly see the improvement in the BER performance for the randomly chosen $\boldsymbol{A}$ matrix. To further improve the BER performance for the SMP-MIMO system, we carefully designed a sparse matrix. The designed sparse matrix is shown in eq.(5.29) and the red line on shows the BER performance for the designed sparse $\boldsymbol{A}$ matrix. We can clearly see that using a carefully designed sparse $\boldsymbol{A}$ matrix further improves the BER performance of the SMP-MIMO system.

Figure 5.10 shows the SER performance of analytical and simulated results for SMP-MIMO. The result shown is for a ($N_T = 4$ and $N_R = 4$) SMP-MIMO system with $\boldsymbol{A}$ matrix of dimension 4x6 as shown in eq.(5.29). In Figure 5.9, the solid line represents the simulated result and the dotted line represents the theoretical result. The solid red line and black dotted line with circles represent a ($N_T = 4$ and $N_R = 4$) SMP-MIMO system with sparse $\boldsymbol{A}$ matrix of 4x6 dimension. We can clearly see that the simulated result almost matches the theoretical results which supports our analysis. Also, the solid blue line and black dotted line with square box show the result for the ($N_T = 5$ and $N_R = 4$) SMP-MIMO system. Here, we can clearly see the SER performance improvement with an increase in the number

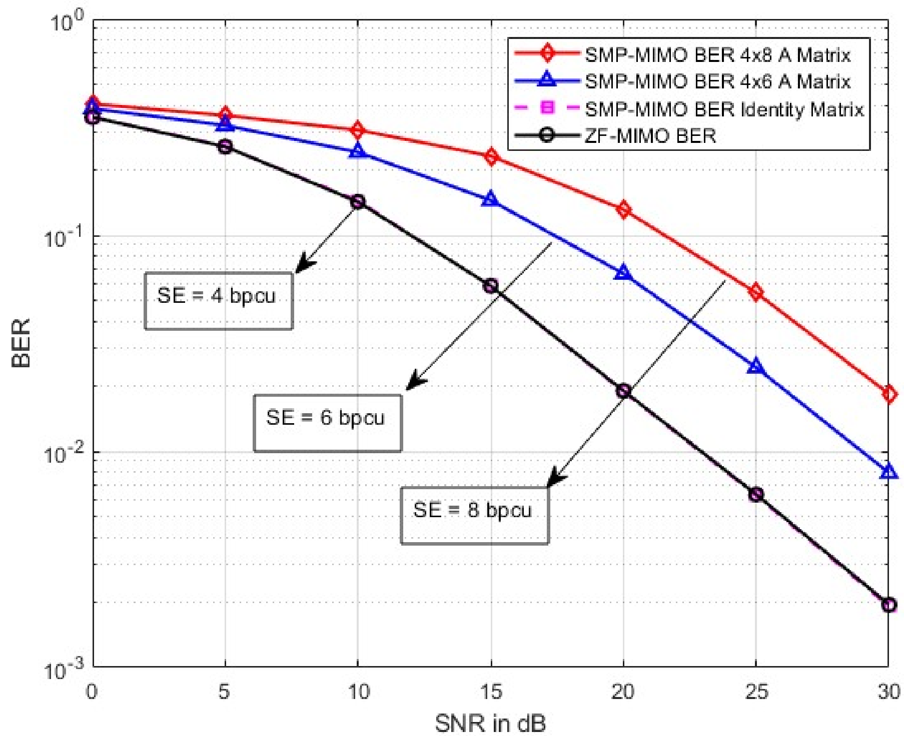

Figure 5.12: BER comparison between SMP-MIMO (proposal two) system with ZF-MIMO system.

of transmit antenna because of the diversity gain.

Figure 5.11 shows the comparison of the spectral efficiency for the proposed SMP-MIMO system with the existing ZF-precoded MIMO system with $N_T = 4$, $N_R = 4$ and $N_T^{'} = 8$. The spectral efficiency for SMP-MIMO is $N_T^{'} \log_2(M)$ bpcu and for ZF-precoded MIMO is $N_R \log_2(M)$ bpcu. The orange line shows the spectral efficiency curve for the SMP-MIMO system and the blue line shows the spectral efficiency curve for the ZF-precoded MIMO system. From Figure 5.10, we can clearly see the massive increase in spectral efficiency with the proposed SMP-MIMO system.

Figure 5.12 shows the BER performance comparison between the proposed SMP-MIMO system and the ZF-precoded MIMO system. The red curve shows the BER performance for the SMP-MIMO system with system parameters ($N_T = 4$, $N_R = 4$, $M = 2$) with sparse $\boldsymbol{A}$ matrix of 4x8 dimension which offers spectral efficiency of 8 bpcu. The blue curve shows the BER performance for the SMP-MIMO system with system parameters ($N_T = 4$, $N_R = 4$, $M = 2$) with sparse $\boldsymbol{A}$ matrix of 4x6 dimension which offers spectral efficiency of 6 bpcu. The magenta curve shows the BER performance for $\boldsymbol{A}$ matrix where $\boldsymbol{A}$ is an Identity matrix of

4x4 dimension, offering spectral efficiency of 4 bpcu. The black curve shows the BER performance for a ZF-precoded MIMO system which also offers the spectral efficiency of 4 bpcu. The magenta curve perfectly overlaps the black curve which is a special case of the SMP-MIMO system. In this special case, if you choose $\boldsymbol{A}$ to be an identity matrix then the SMP-MIMO system will behave as a ZF-precoded MIMO system. The proposed SMP-MIMO system increases the spectral efficiency by a factor of 2 compared to ZF-precoded MIMO without any increase in the number of transmit or receive antenna but at the cost of slightly degraded BER performance. We know that we can further improve the BER performance of SMP-MIMO by increasing the diversity gain.

5.3 Computational Complexity Improvement For Proposal Two

Proposal Two proposed a novel technique of SMP-MIMO. The decoder used at the receiver in SMP-MIMO system is ML decoder. ML decoder is an optimal decoder as it compares all the possible transmitted signals with the received signal, which is basically a brute-force algorithm to estimate the transmitted signal. Now, because of this it has very intense computational complexity which is harder to implement in a real world system. Since we are using sparse matrix as precoder. We can counteract the resulting computational complexity of the SMP-MIMO system by applying a compressive sensing algorithm or a belief propagation algorithm that leverages the sparseness of the stage one precoding matrix.

The idea is to use a LDPC matrix as the stage one sparse matrix precoder. LDPC matrices are sparse matrices containing only a few number of 1's in comparison to the number of 0's in the matrix. There are two types of LDPC matrix.

- **Regular Matrix:** When the parity check matrix is low density and number of 1's in each column of the parity check matrix are constant and similarly number of 1's in each row are also constant then the given parity check matrix is called as Regular LDPC matrix.

- **Irregular Matrix:** When the parity check matrix is low density but the number of 1's in each row or column of the parity check matrix are not constant, then the given parity check matrix is called an Irregular LDPC matrix.

There are two different ways to represent LDPC matrices.

Matrix Representation

As the name suggests, we see the matrix form of LDPC. The matrix shown below is an example of a parity check matrix.

$$\boldsymbol{A}_{\text{LDPC}} = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \end{bmatrix} \tag{5.41}$$

The above parity check matrix is not exactly Low-Density. For a matrix to be low-density, certain conditions are to be satisfied. Let's say there is a matrix of dimension r x c. Here, r denotes the number of rows and c denotes the number of columns in the matrix. p_r denotes the number of 1′s in each row and p_c denotes number of 1′s in each column. The two conditions which are needed to satisfy to be low-density are given below:

- Number of 1′s in each row must be much smaller than the number of rows in the matrix i.e. $p_r << r$.
- Number of 1′s in each column must be much smaller than the number of columns in the matrix i.e. $p_c << c$.

Let′s see another slightly bigger matrix than the above example.

$$
\boldsymbol{A}_{\text{LDPC}} = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\
1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \\
0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\
1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1
\end{bmatrix} \tag{5.42}
$$

The matrix shown above is a 9 x 12 parity check matrix. Here we can see that number of 1′s in each row are four ($p_r = 4$) and number of 1′s in each column are three ($p_c = 3$) which are comparatively smaller than the number of rows ($r = 9$) and number of column ($c = 12$) in the matrix. Hence, this matrix can be called as an LDPC matrix. There are bigger standard LDPC matrices as well. For example, there is an LDPC matrix of dimension 2592 x 5184 where number of 1′s in each row are six $p_r = 6$ and number of 1′s in each column are three $p_c = 3$.

Graphical Representation

The graphical representation of the LDPC matrix is known as Tanner Graph. Tanner graphs are bipartite graphs. Tanner graph is separated into two separate sets. Edges are only connecting nodes between the two different sets. The two types of nodes in a Tanner graph are

- Check Nodes
- Variable Nodes

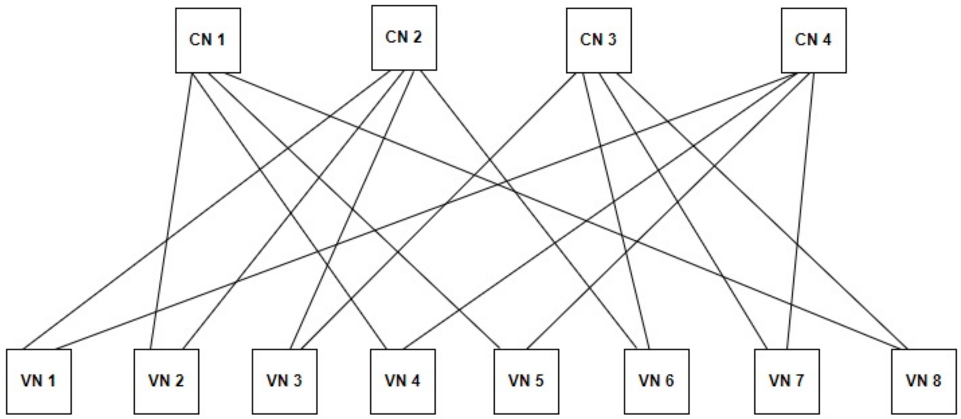

Figure 5.13: Graphical representation of 4x8 parity matrix.

The degree of a node is the number of edges connected to that node. The degree of the check node is denoted by d_c and the degree of the variable node is denoted by d_v. When we perform decoding, beliefs are exchanged between the check node and variable node through the edges. The example of tanner graph representation for a 4x 8 LDPC matrix is shown in Figure 5.13. Here, the value of $d_c = 4$ and $d_v = 2$.

In the SMP-MIMO system, when we use an LDPC as a sparse matrix precoder. We also get the improvement over BER compared to the designed $\boldsymbol{A}$ matrix. Our received vector $\mathbf{y} = \beta_{SMP}\mathbf{Ax} + \mathbf{n}$ which is shown below:

$$\boldsymbol{y} = \beta \begin{bmatrix} 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \mathrm{x}_1 \\ \mathrm{x}_2 \\ \mathrm{x}_3 \\ \mathrm{x}_4 \\ \mathrm{x}_5 \\ \mathrm{x}_6 \\ \mathrm{x}_7 \\ \mathrm{x}_8 \end{bmatrix} + \begin{bmatrix} \mathrm{n}_1 \\ \mathrm{n}_2 \\ \mathrm{n}_3 \\ \mathrm{n}_4 \end{bmatrix} = \beta \begin{bmatrix} \mathrm{x}_2 + x_4 + x_5 + x_8 + n_1 \\ \mathrm{x}_1 + x_2 + x_3 + x_6 + n_2 \\ \mathrm{x}_3 + x_6 + x_7 + x_8 + n_3 \\ \mathrm{x}_1 + x_4 + x_5 + x_7 + n_4 \end{bmatrix}$$

To decode this received vector $\boldsymbol{y}$, before we used ML decoder but now we are proposing a new decoder based on belief propagation which reduces the complexity for the receiver. To further improve the BER performance we use power domain multiplier whose job is to perform one to one mapping between all possible transmit vector.

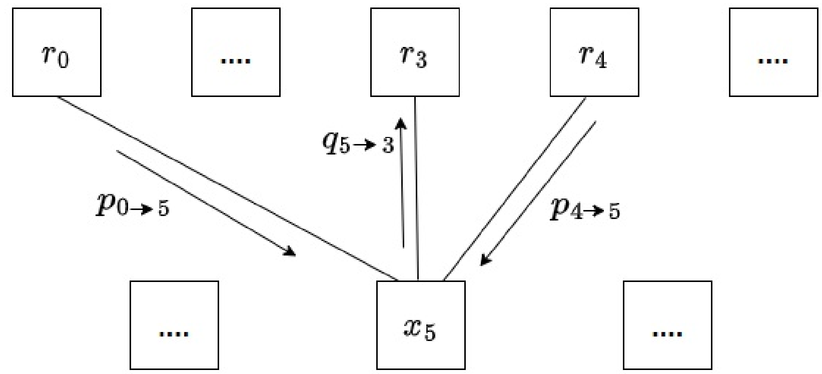

Figure 5.14: Step 1: Update from Variable Node to Check Node.

Belief Propagation based on Independent Probability Evaluation (BP-IPE):

Our goal is to find the information-bearing vector $\boldsymbol{x}$ given the received $\boldsymbol{y}$ vector. Our proposal is to equate the received $\boldsymbol{y}$ vector to the check nodes and the information-bearing vector $\boldsymbol{x}$ to the variable nodes of the tanner graph of $\boldsymbol{A}$ matrix. Variable nodes and check nodes exchange their respective beliefs in regard to the variable nodes { $\boldsymbol{x_m}$} given the received check nodes { $\boldsymbol{y_l}$}. Exchange between the check nodes and variable nodes takes place through the edges of tanner graph of $\boldsymbol{A}$ matrix.

In the first stage, a variable node uses the beliefs received from the other connected check nodes to convey a belief about its value to a check node [26]. The second phase involves a check node sending a belief to a variable node based on the measured value of the check node and the beliefs it has received from the other nodes. These first two steps are repeated.

Derivation Through Example: Let us take an example with $d_c = 3, d_v = 3$ and $\boldsymbol{y}$ = $\boldsymbol{r}$. Let $q_{m\to l}$ and $p_{l\to m}$ both denote the probability $p(\mathbf{x_m} = 1)$. In first step, $q_{m\to l}$ is sent by variable node m to check node l. In second step, $p_{l\to m}$ is sent by check node l to variable node m. The first stage is initiated by each variable node delivering the same message $q_{m\to l} = p$ to each of its associated check nodes where p is already known at the receiver. In the forthcoming iterations, under the assumption that the messages delivered by the different check nodes to a variable node are statistically independent. In Figure 5.14, we can see step 1 which shows varibale node to check node update. Here, the belief sent from variable node 5 to

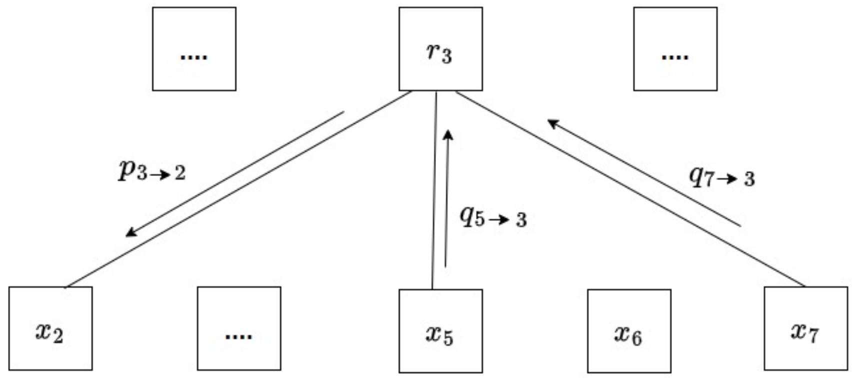

Figure 5.15: Step 2: Update from Check Node to variable Node.

check node 3 is

$$q_{5\to 3} = p_{0\to 5}p_{4\to 5} \tag{5.43}$$

So, $q_{m\to l}$ is simply the product of the beliefs received from the other connecting check nodes. Our key task is to come up with a fresh approach for step two. In Figure 5.15, we can see step 2 which shows check node to varibale node update. Let us understand this with an example. Let's take check node $l = 3$ with a measured value $r_3 = 2$. Three variable nodes connected to this check node are $m = 2, 5, 7$. The variable nodes $m = 5$ and $m = 7$ are taken as independent random variables (RVs) in the proposed BP-IPE approach. There are three events possible for $r_3 = 2$ in step two. All three possible events are shown in table 5.5

Event	$\mathbf{x_5}$	$\mathbf{x_7}$	$\mathbf{x_2}$	$\mathbf{r_3}$
A	0	1	1	2
B	1	0	1	2
C	1	1	0	2

Table 5.5: Three possible events when $r_3 = 2$.

but Since $r_3 = 2$, the variable node 2 takes the value of 1 in one of the possible two events:

- **Event A:** When variable node 5 is 0 and variable node 7 is 1. The probability for event A is equals to $(1 - q_{5\to 3})q_{7\to 3}$

- **Event B:** When variable node 5 is 1 and variable node 7 is 0. The probability for event B is equals to $q_{5\to3}(1-q_{7\to3})$

The event A and B in which variable node 2 equals one is a union of these two mutually-exclusive events. Thus, the required probability is

$$p_{3\to2} = \alpha(1-q_{5\to3})q_{7\to3} + q_{5\to3}(1-q_{7\to3}) \tag{5.44}$$

In the above equation, α is a normalizing constant such that $p_{l\to m}+\mathrm{p}'_{l\to m} = 1$

Simulation Results:

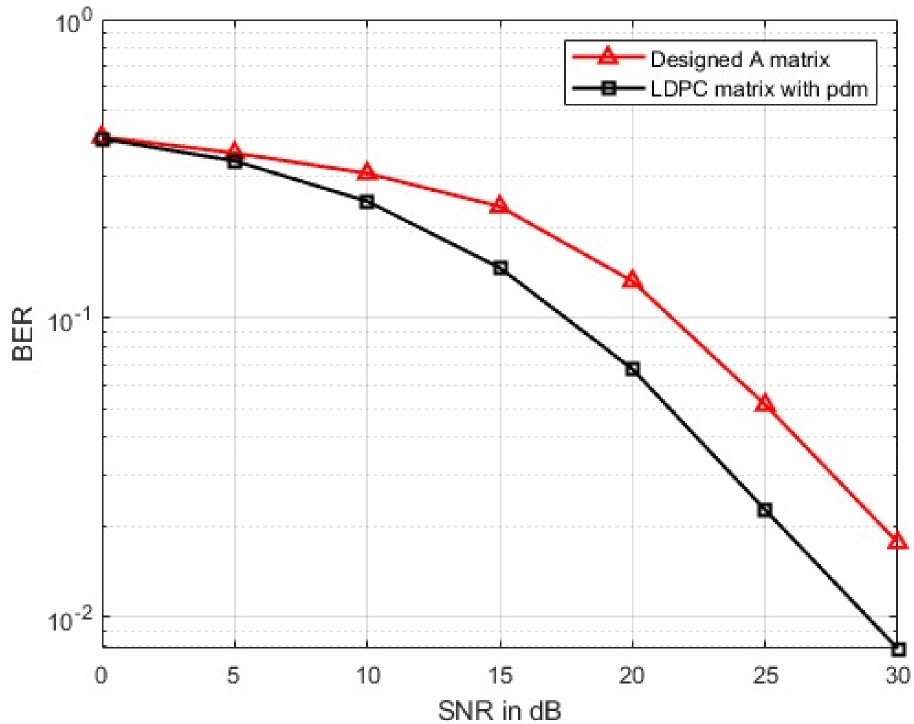

Figure 5.16: BER improvement for SMP-MIMO (proposal two) system with 4x8 LDPC matrix using power domain multiplier.

For simulation, we have considered BPSK and 4-QAM transmission with $N_T = 4$ and $N_R = 4$. We shown further improvement in the BER performance of SMP-MIMO system by using a standard LDPC matrix as $\boldsymbol{A}$ matrix. We have shown the simulated results for BER performance with reduced complexity at the receiver of the SMP-MIMO system using the BP-IPE algorithm. We performed 10000 Monte Carlo simulations to obtain the following results.

Figure 5.16 shows the BER performance improvement for the SMP-MIMO system. The black curve shows the BER performance of the SMP-MIMO system with

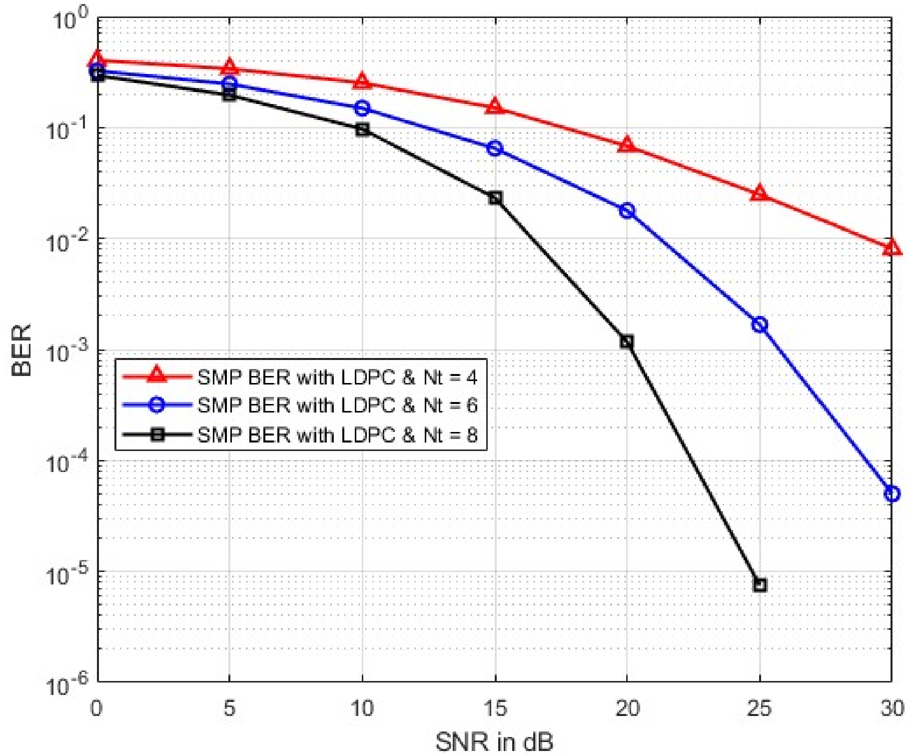

Figure 5.17: BER Performance of SMP-MIMO (proposal two) using 4x8 LDPC matrix with power domain multiplier with $N_T = 4, 6, 8$.

4x8 LDPC matrix using power domain multiplier shown in eq.(5.39). The red curve shows the BER performance for the earlier designed 4x8 matrix. We can clearly see that when we use a standard LDPC matrix with power domain multiplier for the $\boldsymbol{A}$ matrix the BER performance improves compared to the earlier designed $\boldsymbol{A}$ matrix.

Figure 5.17 shows the BER performance with different system parameters. We fixed the number of receive antenna, the APM constellation size and $\boldsymbol{A}$ matrix used is standard LDPC matrix multiplied with power domain multiplier. We varied the number of transmit antenna. The red curve shows the BER performance of the SMP-MIMO system with the number of transmit antennas ($N_T = 4$). The blue curve shows the BER performance of the SMP-MIMO system with the number of transmit antennas ($N_T = 6$). The black curve shows the BER performance of the SMP-MIMO system with the number of transmit antennas ($N_T = 8$). We can clearly see that as we go on increasing the number of transmit antennas keeping other parameters fixed we see a great improvement in the BER performance because of the increased value of β. However, the study for the exact improvement of 8×4 over the 6×4 and 4×4 system is still pending.

CHAPTER 6
Conclusions

We proposed a novel technique of Sparse Matrix Precoded-MIMO (SMP-MIMO) system. In proposal one, by virtue of the sparse $\boldsymbol{A}$ matrix, we are logarithmically increasing the spectral efficiency. The spectral efficiency of proposal one is

$$SE_{\text{SMP-PSM}} = \log_2(N_T^{'}) + \log_2(M) \tag{6.1}$$

where $N_T^{'}$ » N_T and N_R. Therefore, increasing the spectral efficiency of proposal one beyond the existing spectral efficiency of the PSM-MIMO system. We have used different $\boldsymbol{A}$ matrices for this objective and found out that we are getting the best result by the carefully designed $\boldsymbol{A}$ matrix. At the receiver, we have performed ML decoding. We have also shown the simulation results for proposal one. Further, we can use a belief propagation algorithm-based receiver to further reduce the computational complexity of the receiver.

The proposal two increases the spectral efficiency beyond the existing spectral efficiency of the MIMO system. By virtue of the sparse matrix precoder, we are able to provide a multiplicative increase in spectral efficiency. The multiplicative factor providing the increase in the spectral efficiency is $N_T^{'}$. Spectral efficiency for the SMP-MIMO system is

$$SE_{\text{SMP}} = N_T^{'} \log_2(M) \tag{6.2}$$

where $N_T^{'}$ » N_T and N_R. Therefore, allowing us to increase the spectral efficiency of the proposed SMP-MIMO system compared to the spectral efficiency of the MIMO system without any increase in the number of transmit antenna or receive antenna and no increase in the size of the APM constellation. We have done the theoretical analysis for the SMP-MIMO system. Our simulation results supports our analysis for a given $\boldsymbol{A}$ matrix. At the receiver, we have used an ML decoder. ML detector has very high computational complexity. To reduce the computational complexity, we have leveraged the sparseness of the $\boldsymbol{A}$ matrix and designed a belief propagation algorithm-based receiver which is practical to implement in a real-world system. We have shown simulated results for the SMP-MIMO system with an ML receiver. We have also proposed the theory for the belief propagation algorithm-based receiver.

CHAPTER 7

Future Work

In Chapter 5, we used ML receiver and proposed theory of a new BP-IPE algorithm-based receiver. We further need to simulate the belief propagation receiver. We can also look for other receivers possible for the proposed schemes. In SMP-MIMO, we can use other compressive sensing techniques at the receiver to improve our results further. For proposal one, we could work on the belief propagation-based receiver. The proposed work can be extended to massive MIMO systems as well.

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